Question			Answer	Marks	Guidance
1	(i)		<table><tr><td>x</td><td>0</td><td>0.1963</td><td>0.3927</td><td>0.5890</td><td>0.7854</td></tr><tr><td>y</td><td>0</td><td>0.4493</td><td>0.6792</td><td>0.9498</td><td>1.3254</td></tr></table>	B2,1,0	For values 0.4493,0.6792,0.9498 (**4dp** or better soi) [accept truncated to 4 figs after dec point] [cannot assume values of form $(\pi/16)^3 + \sqrt{(\sin \pi/16)}$ are correct unless followed by correct total at some later stage as some will be in degree mode]
			$A = (\pi/32)\,[(0 + 1.3254) + 2(0.4493 + 0.6792 + 0.9498)]$	M1	Use of the trapezium rule. Trapezium rule formula for **4 strips** must be seen, with or without substitution seen. **Correct *h* must be soi.** [accept separate trapezia added]
			$= 0.538$	A1	0.538 **www 3dp only** (NB using 1.325 is ww) SC B0 0.538 without any working as no indication of strips or method used SC B1 0.538 with some indication of 4 strips but no values seen Correct values followed by 0.538 scores B2 B0 Correct values followed by correct formula for 4 strips, with or without substitution seen, then A= 0.538 scores 4/4. Correct formula for 4 strips and values of form $((\pi/16)^3 + \sqrt{(\sin\pi/16)}$… followed by correct answer scores 4/4 (or ¾ with wrong dp) **NB Values given in the table to only 3dp give apparently the correct answer, but scores B0,M1A0 ww**
				[4]	
1	(ii)		Not possible to say, eg some trapezia are above and some below curve oe.	B1	Need a reason. Must be without further calculation.
				[1]	

2(i) When $x = 0.5$, $y = 1.1180$ $\Rightarrow$ $A \approx 0.25/2\{1+1.4142+2(1.0308+1.1180+1.25)\}$ $= 0.25 \times 4.6059 = 1.151475$ $= 1.151$ (3 d.p.)*	B1 M1 E1 [3]	4dp (0.125 x 9.2118) need evidence
(ii) Explain that the area is an over-estimate. *or* The curve is below the trapezia, so the area is an over- estimate. This becomes less with more strips. o*r* Greater number of strips improves accuracy so becomes less	B1 B1 [2]	or use a diagram to show why
(iii) $V = \int_0^1 \pi y^2 dx$ $= \int_0^1 \pi(1+x^2)dx$ $= \pi\left[(x+x^3/3)\right]_0^1$ $= 1\frac{1}{3}\pi$	 M1 B1 A1 [3]	 allow limits later $x + x^3/3$ exact

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3	(a)		$V = \int_0^2 \pi y^2 \mathrm{d}x = \int_0^2 \pi(1 + \mathrm{e}^{2x})\mathrm{d}x$	M1	$\int_0^2 \pi(1 + \mathrm{e}^{2x})\mathrm{d}x$ limits must appear but may be later condone omission of *dx* if intention clear
			$= \pi\left[x + \frac{1}{2}\mathrm{e}^{2x}\right]_0^2$	B1	$\left[x + \frac{1}{2}\mathrm{e}^{2x}\right]$ independent of π and limits
			$= \pi(2 + ½\ \mathrm{e}^4 - ½\ )$	DM1	dependent on first M1.Need **both** limits substituted in their integral of the form $ax + b\,e^{2x}$, where a ,b non-zero constants. Accept answers including e^0 for M1. Condone absence of π for M1 at this stage
			$= ½\ \pi(3 + \mathrm{e}^4)$	A1 **[4]**	cao exact only
3	(b)	(i)	$x = 0$, $y = 1.4142$; $x = 2$, $y = 7.4564$	B1	1.414, 7.456 or better
			$A = 0.5/2\{(1.4142 + 7.4564) + 2(1.9283 + 2.8964 + 4.5919)\}$	M1	correct formula seen (can be implied by correct intermediate step eg 27.7038../4)
			$= 6.926$	A1 **[3]**	6.926 or 6.93 (do not allow more dp)
3	(b)	(ii)	8 strips: 6.823, 16 strips: 6.797		
			Trapezium rule overestimates this area, but the overestimate gets less as the no of strips increases.	B1 **[1]**	oe

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4	(i)		1, 0.6186, 0 $A \approx (\pi/16)\{1 + 0+2(0.9612+0.8409+0.6186)\}$ $= 1.147$ (3 dp)	B1 M1 A1 **[3]**	4dp (or more) ft their table. Need to see trapezium rule. cao
4	(ii)		The estimate will increase, because the trapezia will be below but closer to the curve, reducing the error.	B1 **[1]**	o.e., or an illustration using the curve full answer required

5(i) x | - | -1 | 0 | 1 | 2 y | 1.0655 | 1.1696 | 1.4142 | 1.9283 | 2.8964 $A \approx$ ½×1{1.0655+2.8964+2(1.1696+1.4142+1.9283)} = 6.493	B2,1,0 M1 A1 [4]	table values formula 6.5 or better www
(ii) Smaller, as the trapezium rule is an over-estimate in this case and the error is less with more strips	B1 B1 [2]	

6(i) $A \approx 0.5[\frac{(1.1696+1.0655}{2}+1.1060]$ = 1.11 (3 s f.)	M1 A1 cao [2]	Correct expression for trapezium rule
(ii) $(1+e^{-x})^{1/2} = 1+\frac{1}{2}e^{-x}+\frac{\frac{1}{2}\cdot-\frac{1}{2}}{2!}(e^{-x})^2+...$ $\approx 1+\frac{1}{2}e^{-x}-\frac{1}{8}e^{-2x}$ *	M1 A1 E1 [3]	Binomial expansion with p = ½ Correct coeffs
(iii) $I = \int_1^2 (1+\frac{1}{2}e^{-x}-\frac{1}{8}e^{-2x})dx$ $= \left[x-\frac{1}{2}e^{-x}+\frac{1}{16}e^{-2x}\right]_1^2$ $= (2-\frac{1}{2}e^{-2}+\frac{1}{16}e^{-4})-(1-\frac{1}{2}e^{-1}+\frac{1}{16}e^{-2})$ = 1.9335 − 0.8245 = 1.11 (3 s.f.)	M1 A1 A1 [3]	integration substituting limits into correct expression