1 Using partial fractions, find $\displaystyle\int \frac{x}{(x+1)(2x+1)}\,dx$. **[7]**

2 **(i)** Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute, expressing α in terms of π. **[4]**

(ii) Write down the derivative of $\tan\theta$.

Hence show that $\displaystyle\int_0^{\frac{1}{3}\pi} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}\,d\theta = \frac{\sqrt{3}}{4}$. **[4]**

3 In a chemical process, the mass M grams of a chemical at time t minutes is modelled by the differential equation

$$\frac{dM}{dt} = \frac{M}{t(1+t^2)}.$$

(i) Find $\int \frac{t}{1+t^2}\,dt$. [3]

(ii) Find constants A, B and C such that

$$\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}.$$ [5]

(iii) Use integration, together with your results in parts **(i)** and **(ii)**, to show that

$$M = \frac{Kt}{\sqrt{1+t^2}},$$

where K is a constant. [6]

(iv) When $t = 1$, $M = 25$. Calculate K.

What is the mass of the chemical in the long term? [4]

4 The growth of a tree is modelled by the differential equation

$$10\frac{\mathrm{d}h}{\mathrm{d}t} = 20 - h,$$

where h is its height in metres and the time t is in years. It is assumed that the tree is grown from seed, so that $h = 0$ when $t = 0$.

(i) Write down the value of h for which $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$, and interpret this in terms of the growth of the tree. **[1]**

(ii) Verify that $h = 20\left(1 - \mathrm{e}^{-0.1t}\right)$ satisfies this differential equation and its initial condition. **[5]**

The alternative differential equation

$$200\frac{\mathrm{d}h}{\mathrm{d}t} = 400 - h^2$$

is proposed to model the growth of the tree. As before, $h = 0$ when $t = 0$.

(iii) Using partial fractions, show by integration that the solution to the alternative differential equation is

$$h = \frac{20\left(1 - \mathrm{e}^{-0.2t}\right)}{1 + \mathrm{e}^{-0.2t}}.$$

[9]

(iv) What does this solution indicate about the long-term height of the tree? **[1]**

(v) After a year, the tree has grown to a height of 2 m. Which model fits this information better? **[3]**

5 **(i)** Find the first three non-zero terms of the binomial expansion of $\frac{1}{\sqrt{4 \quad x^2}}$ for $|x| < 2$. [4]

(ii) Use this result to find an approximation for $\int_0^1 \frac{1}{\sqrt{4 \quad x^2}}\mathrm{d}x$, rounding your answer to 4 significant figures. [2]

(iii) Given that $\int \frac{1}{\sqrt{4 \quad x^2}}\mathrm{d}x = \arcsin\left(\frac{1}{2}x\right) + c$, evaluate $\int_0^1 \frac{1}{\sqrt{4 \quad x^2}}\mathrm{d}x$, rounding your answer to 4 significant figures. [1]