1 $\frac{x}{(x+1)(2x+1)} = \frac{A}{x+1} + \frac{B}{(2x+1)}$ $\Rightarrow$ $x = A(2x+1) + B(x+1)$ $x = -1 \Rightarrow -1 = -A \Rightarrow A = 1$ $x = -½ \Rightarrow -½ = ½ B \Rightarrow B = -1$ $\Rightarrow$ $\frac{x}{(x+1)(2x+1)} = \frac{1}{x+1} - \frac{1}{(2x+1)}$ $\Rightarrow$ $\int \frac{x}{(x+1)(2x+1)}dx = \int \frac{1}{x+1} - \frac{1}{(2x+1)}dx$ $= \ln(x+1) - ½\ln(2x+1) + c$	M1 M1 A1 A1 B1 B1 A1 [7]	correct partial fractions substituting, equating coeffts or cover-up $A = 1$ $B = -1$ ln(x + 1) ft their A -½ ln(2x + 1) ft their B cao – must have c

2(i) $\cos\theta + \sqrt{3}\sin\theta = r\cos(\theta - \alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = \sqrt{3}$ $\Rightarrow R^2 = 1^2 + (\sqrt{3})^2 = 4, R = 2$ $\tan\alpha = \sqrt{3}$ $\Rightarrow \alpha = \pi/3$	B1 M1 M1 A1 [4]	$R = 2$ equating correct pairs $\tan\alpha = \sqrt{3}$ o.e.
(ii) derivative of $\tan\theta$ is $\sec^2\theta$ $\int_0^{\frac{\pi}{3}} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}d\theta = \int_0^{\frac{\pi}{3}} \frac{1}{4}\sec^2(\theta - \frac{\pi}{3})d\theta$ $= \left[\frac{1}{4}\tan(\theta - \frac{\pi}{3})\right]_0^{\frac{\pi}{3}}$ $= ¼(0 - (-\sqrt{3}))$ $= \sqrt{3}/4$ *	B1 M1 A1 E1 [4]	 ft their α $\frac{1}{R^2}$ [tan ($\theta - \pi/3$)] ft their R,α(in radians) www

3 (i) $\int \frac{t}{1+t^2} dt = ½ \ln(1 + t^2) + c$ **OR** $\int \frac{t}{1+t^2} dt$ let $u = 1 + t^2$, $du = 2t dt$ $= \int \frac{1/2}{u} du$ $= ½ \ln u + c$ $= ½ \ln(1 + t^2) + c$	**M1** **A2** **M1** **A1** **A1** **[3]**	$k \ln(1 + t^2)$ $½ \ln(1 + t^2)$ [$+c$] substituting $u = 1 + t^2$ condone no c
(ii) $\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}$ $\Rightarrow$ $1 = A(1+ t^2) + (Bt + C)t$ $t = 0 \Rightarrow 1 = A$ coeff[t] of t^2 $\Rightarrow 0 = A + B$ $\Rightarrow B = -1$ coeff[t] of t $\Rightarrow 0 = C$ $\Rightarrow$ $\frac{1}{t(1+t^2)} = \frac{1}{t} - \frac{t}{1+t^2}$	 **M1** **M1** **A1** **A1** **A1** **[5]**	 Equating numerators substituting or equating coeff[t]'s dep 1[st] M1 $A = 1$ $B = -1$ $C = 0$
(iii) $\frac{dM}{dt} = \frac{M}{t(1+t^2)}$ $\Rightarrow$ $\int \frac{1}{M} dM = \int \frac{1}{t(1+t^2)} dt = \int [\frac{1}{t} - \frac{t}{1+t^2}] dt$ $\Rightarrow$ $\ln M = \ln t - ½ \ln(1 + t^2) + c$ $= \ln(\frac{e^c t}{\sqrt{1+t^2}})$ $\Rightarrow$ $M = \frac{Kt}{\sqrt{1+t^2}}$ * where $K = e^c$	 **M1** **B1** **A1ft** **M1** **M1** **E1** **[6]**	 Separating variables and substituting their partial fractions $\ln M = \ldots$ $\ln t - ½ \ln(1 + t^2) + c$ combining $\ln t$ and $½ \ln(1 + t^2)$ $K = e^c$ o.e.
(iv) $t = 1$, $M = 25 \Rightarrow 25 = K/\sqrt{2}$ $\Rightarrow$ $K = 25\sqrt{2} = 35.36$ As $t \to \infty$, $M \to K$ So long term value of M is 35.36 grams	**M1** **A1** **M1** **A1ft** **[4]**	 $25\sqrt{2}$ or 35 or better soi ft their K.

4	**(i)**		h = 20, stops growing	B1 **[1]**	**AG** need interpretation	
	(ii)		$h = 20 - 20e^{-t/10}$ $dh/dt = 2e^{-t/10}$ $20e^{-t/10} = 20 - 20(1 - e^{-t/10}) = 20 - h$ $= 10 dh/dt$ when $t = 0$, $h = 20(1 - 1) = 0$ ……………………………………………………… **OR** verifying by integration $\int \frac{dh}{20-h} = \int \frac{dt}{10}$ $\Rightarrow -\ln(20-h) = 0.1t + c$ $h = 0, t = 0, \Rightarrow c = -\ln 20$ $\Rightarrow \ln(20-h) = -0.1t + \ln 20$ $\Rightarrow \ln(\frac{20-h}{20}) = -0.1t$ $\Rightarrow 20 - h = 20e^{-0.1t}$ $\Rightarrow h = 20(1 - e^{-0.1t})$	M1A1 M1 A1 B1 M1 A1 B1 M1 A1 **[5]**	differentiation (for M1 need $ke^{-t/10}$, k const) oe eg $20 - h = 20 - 20(1 - e^{-t/10}) = 20e^{-t/10}$ $= 10dh/dt$ (showing sides equivalent) initial conditions …………………………………………………………………………….. sep correctly and intending to integrate correct result (condone omission of c, although no further marks are possible) condone ln (h – 20) as part of the solution at this stage constant found from expression of correct form (at any stage) but B0 if say $c = \ln(-20)$ (found using $\ln(h - 20)$) combining logs and anti-logging (correct rules) correct form (do not award if B0 above)	

Question			Answer	Marks	Guidance
4	**(iii)**		$\frac{200}{(20+h)(20-h)} = \frac{A}{20+h} + \frac{B}{20-h}$		
			$\Rightarrow \quad 200 = A(20 - h) + B(20 + h)$	M1	cover up, substitution or equating coeffs
			$h = 20 \Rightarrow 200 = 40\,B,\ B = 5$	A1	
			$h = -20 \Rightarrow 200 = 40A\ ,\ A = 5$	A1	
			$200\ dh/dt = 400 - h^2$		
			$\Rightarrow \quad \int \frac{200}{400-h^2}\,dh = \int dt$	M1	separating variables and intending to integrate (condone sign error)
			$\Rightarrow \quad \int (\frac{5}{20+h} + \frac{5}{20-h})\,dh = \int dt$		substituting partial fractions
			$\Rightarrow \quad 5\ln(20 + h) - 5\ln(20 - h) = t + c$	A1	ft their A,B, condone absence of c, Do not allow ln(h-20) for A1.
			When $t = 0$, $h = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$	B1	cao need to **show** this. c can be found at any stage. **NB c = ln(–1)** (from ln(h –20)) or similar **scores B0.**
			$\Rightarrow \quad 5\ln\frac{20+h}{20-h} = t$		
			$\Rightarrow \quad \frac{20+h}{20-h} = e^{t/5}$	M1	anti-logging an equation of the correct form . Allow if c = 0 clearly stated (provided that c = 0) even if B mark is not awarded, but do not allow if c omitted. Can ft their c.
			$\Rightarrow \quad 20 + h = \quad - h)e^{t/5} = 20 \quad {}^{t/5} - h\ e^{t/5}$		
			$\Rightarrow \quad h + h\ e^{t/5} = 20\ e^{t/5} - 20$		
			$\Rightarrow \quad h(e^{t/5} + 1) = 20(e^{t/5} - 1)$	DM1	maki h the subject, dependent on previous mark
			$\Rightarrow \quad h = \frac{20(e^{t/5}-1)}{e^{t/5}+1}$		**NB** method marks can be in either order, in which case the dependence is the other way around.(In which case, 20 + h is divided by 20 – h first to isolate h).
			$\Rightarrow \quad h = \frac{20(1-e^{-t/5})}{1+e^{-t/5}}\ *$	A1	**AG must have obtained B1** (for c) in order to obtain final A1**.**
				[9]	
4	**(iv)**		As $t \to \infty$, $h \to 20$. So long-term height is 20m.	B1 **[1]**	www
4	**(v)**		1st model $h = 20(1 - e^{-0\ 1}) = 1.90..$	B1	O 1st model h = 2 gives t = 1.05..
			2nd model $h = 20(e^{1/5} - 1)/(e^{1/5} + 1) = 1.99..$	B1	2nd model h = 2 gives t = 1.003..
			so 2nd model fits data better	B1 dep **[3]**	dep previous B1s correct

5 (i) $\frac{1}{\sqrt{4-x^2}} = 4^{-\frac{1}{2}}(1-\frac{1}{4}x^2)^{-\frac{1}{2}}$	M1	
$= \frac{1}{2}[1+(-\frac{1}{2})(-\frac{1}{4}x^2)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{1}{4}x^2)^2+...]$	M1 A1	Binomial coeffs correct Complete correct expression inside bracket
$= \frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+...$	A1	cao
(ii) $\int_0^1 \frac{1}{\sqrt{4-x^2}}dx \approx \int_0^1 (\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4)dx$ $= \left[\frac{1}{2}x+\frac{1}{48}x^3+\frac{3}{1280}x^5\right]_0^1$ $= \frac{1}{2}+\frac{1}{48}+\frac{3}{1280}$ = 0.5232 (to 4 s.f.)	M1ft A1	
(iii) $\int_0^1 \frac{1}{\sqrt{4-x^2}}dx = \left[\arcsin\frac{x}{2}\right]_0^1$ = π/6 = 0.5236	B1 [7]	