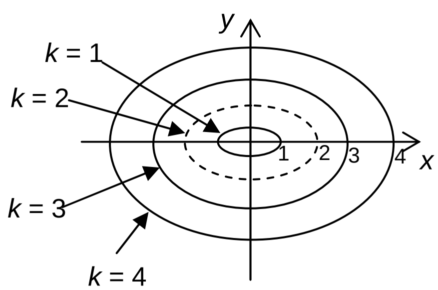
1	**(i)**	$\frac{dx}{dt}=-1(1+e^{-t})^{-2}.-e^{-t}$ $=\frac{e^{-t}}{(1+e^{-t})^2}$ $1-x=1-\frac{1}{1+e^{-t}}$ $1-x=\frac{1+e^{-t}-1}{1+e^{-t}}=\frac{e^{-t}}{1+e^{-t}}$ $\Rightarrow\quad x(1-x)=\frac{1}{1+e^{-t}}\frac{e^{-t}}{1+e^{-t}}=\frac{e^{-t}}{(1+e^{-t})^2}$ $\Rightarrow\quad \frac{dx}{dt}=x(1-x)$ When $t = 0$, $x=\frac{1}{1+e^0}=0.5$	M1 A1 M1 A1 E1 B1 **[6]**	chain rule substituting for $x(1-x)$ $1-x=\frac{1+e^{-t}-1}{1+e^{-t}}=\frac{e^{-t}}{1+e^{-t}}$ [OR,M1 A1 for solving differential equation for t, B1 use of initial condition, M1 A1 making x the subject, E1 required form]
	(ii)	$\frac{1}{(1+e^{-t})}=\frac{3}{4}$ $\Rightarrow\quad e^{-t} = 1/3$ $\Rightarrow\quad t = -\ln 1/3 = 1.10$ years	M1 M1 A1 **[3]**	 correct log rules
	(iii)	$\frac{1}{x^2(1-x)}=\frac{A}{x^2}+\frac{B}{x}+\frac{C}{1-x}$ $\Rightarrow\quad 1 = A(1-x) + Bx(1-x) + Cx^2$ $x = 0 \Rightarrow A = 1$ $x = 1 \Rightarrow C = 1$ coefft of x^2: $0 = -B + C \Rightarrow B = 1$	 M1 M1 B(2,1,0) **[4]**	 clearing fractions substituting or equating coeffs for A,B or C $A = 1$, $B = 1$, $C = 1$ www
	(iv)	$\int\frac{dx}{x^2(1-x)}dx=\int dt$ $\Rightarrow\quad t=\int(\frac{1}{x^2}+\frac{1}{x}+\frac{1}{1-x})dx$ $= -1/x + \ln x - \ln(1-x) + c$ When $t = 0$, $x = ½ \Rightarrow 0 = -2 + \ln ½ - \ln ½ + c$ $\Rightarrow\quad c = 2.$ $\Rightarrow\quad t = -1/x + \ln x - \ln(1-x) + 2$ $=2+\ln\frac{x}{1-x}-\frac{1}{x}$ *	M1 B1 B1 M1 E1 **[5]**	separating variables $-1/x + \ldots$ $\ln x - \ln(1-x)$ ft their A,B,C substituting initial conditions
	(v)	$t=2+\ln\frac{3/4}{1-3/4}-\frac{1}{3/4}=\ln 3+\frac{2}{3}=1.77$ yrs	M1A1 **[2]**	

2(i) (*A*) 9 / 1.5 = 6 hours (*B*) 1 /1.5 = 12 hours	B1 B1 [2]	
(ii) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$ $\Rightarrow \int \frac{d\theta}{\theta - \theta_0} = \int -kdt$ $\Rightarrow \ln(\theta - \theta_0) = -kt + c$ $\theta - \theta_0 = e^{-kt+c}$ $\theta = \theta_0 + Ae^{-kt}$ *	 M1 A1 A1 M1 E1 [5]	 separating variables $\ln(\theta - \theta_0)$ $-kt + c$ anti-logging correctly(with *c*) $A=e^{c}$
(iii) $98 = 50 + Ae^0$ $\Rightarrow A = 48$ Initially $\frac{d\theta}{dt} = -k(98 - 50) = -48k = -1.5$ $\Rightarrow k = 0.03125$*	M1 A1 M1 E1 [4]	
(iv) (*A*) $89 = 50 + 48e^{-0\ 03125t}$ $\Rightarrow 39/48 = e^{-0\ 03125t}$ $\Rightarrow t = \ln(39/48)/(-0.03125) = 6.64$ hours (*B*) $80 = 50 + 48e^{-0\ 03125t}$ $\Rightarrow 30/48 = e^{-0\ 03125t}$ $\Rightarrow t = \ln(30/48)/(-0.03125) = 15$ hours	M1 M1 A1 M1 A1 [5]	equating taking lns correctly for either
(v) Models disagree more for greater temperature loss	B1 [1]	

3 (i) $x = a(1 + kt)^{-1}$ $\Rightarrow$ $dx/dt = -ka(1 + kt)^{-2}$ $= -ka(x/a)^2$ $= -kx^2/a$ * OR $kt = a/x - 1$, $t = a/kx - 1/k$ $dt/dx = -a/kx^2$ $\Rightarrow dx/dt = -kx^2/a$	M1 A1 E1 [3] M1 A1 E1 [3]	Chain rule (or quotient rule) Substitution for x
(ii) When $t = 0$, $x = a \Rightarrow a = 2.5$ When $t = 1$, $x = 1.6 \Rightarrow 1.6 = 2.5/(1 + k)$ $\Rightarrow$ $1 + k = 1.5625$ $\Rightarrow$ $k = 0.5625$	B1 M1 A1 [3]	$a = 2.5$
(iii) In the long term, $x \to 0$	B1 [1]	or, for example, they die out.
(iv) $\frac{1}{2y-y^2} = \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{2-y}$ $\Rightarrow$ $1 = A(2 - y) + By$ $y = 0 \Rightarrow 2A = 1 \Rightarrow A = ½$ $y = 2 \Rightarrow 1 = 2B \Rightarrow B = ½$ $\Rightarrow \frac{1}{2y-y^2} = \frac{1}{2y} + \frac{1}{2(2-y)}$	M1 M1 A1 A1 [4]	partial fractions evaluating constants by substituting values, equating coefficients or cover-up
(v) $\int \frac{1}{2y-y^2} dy = \int dt$ $\Rightarrow$ $\int [\frac{1}{2y} + \frac{1}{2(2-y)}] dy = \int dt$ $\Rightarrow$ $½ \ln y - ½ \ln(2 - y) = t + c$ When $t = 0$, $y = 1 \Rightarrow 0 - 0 = 0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln y - \ln(2 - y) = 2t$ $\Rightarrow$ $\ln \frac{y}{2-y} = 2t$ * $\frac{y}{2-y} = e^{2t}$ $\Rightarrow$ $y = 2e^{2t} - ye^{2t}$ $\Rightarrow$ $y + ye^{2t} = 2e^{2t}$ $\Rightarrow$ $y(1 + e^{2t}) = 2e^{2t}$ $\Rightarrow$ $y = \frac{2e^{2t}}{1+e^{2t}} = \frac{2}{1+e^{-2t}}$ *	M1 B1 ft A1 E1 M1 DM1 E1 [7]	Separating variables ½ ln y – ½ ln(2 – y) ft their A,B evaluating the constant Anti-logging Isolating y
(vi) As $t \to \infty$ $e^{-2t} \to 0 \Rightarrow y \to 2$ So long term population is 2000	B1 [1]	or $y = 2$

4(i) $\cos\theta = \frac{x}{k}, \sin\theta = \frac{2y}{k}$ $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow \left(\frac{x}{k}\right)^2 + \left(\frac{2y}{k}\right)^2 = 1$ $\Rightarrow \frac{x^2}{k^2} + \frac{4y^2}{k^2} = 1$ $\Rightarrow x^2 + 4y^2 = k^2$ *	M1 M1 E1 [3]	Used substitution
(ii) $\frac{dx}{d\theta} = -k\sin\theta, \frac{dy}{d\theta} = \frac{1}{2}k\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\frac{1}{2}k\cos\theta}{k\sin\theta}$ $= -\frac{1}{2}\cot\theta$ $-\frac{x}{4y} = -\frac{2k\cos\theta}{4k\sin\theta} = -\frac{1}{2}\cot\theta = \frac{dy}{dx}$	M1 A1 E1	 oe
or, by differentiating implicitly $2x + 8y\, dy/dx = 0$ $\Rightarrow dy/dx = -2x/8y = -x/4y$*	M1 A1 E1 [3]	
(iii) $k = 2$	B1 [1]	
(iv) 	B1 B1 B1 [3]	1 correct curve –shape and position 2 or more curves correct shape- in concentric form all 3 curves correct
(v) grad of stream path = –1/grad of contour $\Rightarrow \frac{dy}{dx} = -\frac{1}{(-x/4y)} = \frac{4y}{x}$ *	M1 E1 [2]	
(vi) $\frac{dy}{dx} = \frac{4y}{x} \Rightarrow \int\frac{dy}{y} = \int\frac{4dx}{x}$ $\Rightarrow \ln y = 4\ln x + c = \ln e^c x^4$ $\Rightarrow y = Ax^4$ where $A = e^c$. When $x = 2, y = 1 \Rightarrow 1 = 16A \Rightarrow A = 1/16$ $\Rightarrow y = x^4/16$ *	M1 A1 M1 M1 A1 E1 [6]	Separating variables $\ln y = 4\ln x\ (+c)$ antilogging correctly (at any stage) substituting $x = 2, y = 1$ evaluating a correct constant www