1 (i) $\int \frac{t}{1+t^2}dt = ½ \ln(1 + t^2) + c$ **OR** $\int \frac{t}{1+t^2}dt$ let $u = 1 + t^2$, $du = 2tdt$ $= \int \frac{1/2}{u}du$ $= ½ \ln u + c$ $= ½ \ln(1 + t^2) + c$	**M1** **A2** **M1** **A1** **A1** **[3]**	$k \ln(1 + t^2)$ $½ \ln(1 + t^2)$ [$+c$] substituting $u = 1 + t^2$ condone no c
(ii) $\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}$ $\Rightarrow 1 = A(1+ t^2) + (Bt + C)t$ $t = 0 \Rightarrow 1 = A$ coeff[t] of t^2 $\Rightarrow 0 = A + B$ $\Rightarrow B = -1$ coeff[t] of t $\Rightarrow 0 = C$ $\Rightarrow \frac{1}{t(1+t^2)} = \frac{1}{t} - \frac{t}{1+t^2}$	**M1** **M1** **A1** **A1** **A1** **[5]**	Equating numerators substituting or equating coeff[t]s dep 1[st] M1 $A = 1$ $B = -1$ $C = 0$
(iii) $\frac{dM}{dt} = \frac{M}{t(1+t^2)}$ $\Rightarrow \int \frac{1}{M}dM = \int \frac{1}{t(1+t^2)}dt = \int [\frac{1}{t} - \frac{t}{1+t^2}]dt$ $\Rightarrow \ln M = \ln t - ½ \ln(1 + t^2) + c$ $= \ln(\frac{e^c t}{\sqrt{1+t^2}})$ $\Rightarrow M = \frac{Kt}{\sqrt{1+t^2}}$ * where $K = e^c$	**M1** **B1** **A1ft** **M1** **M1** **E1** **[6]**	Separating variables and substituting their partial fractions $\ln M = \ldots$ $\ln t - ½ \ln(1 + t^2) + c$ combining $\ln t$ and $½ \ln(1 + t^2)$ $K = e^c$ o.e.
(iv) $t = 1$, $M = 25 \Rightarrow 25 = K/\sqrt{2}$ $\Rightarrow K = 25\sqrt{2} = 35.36$ As $t \to \infty$, $M \to K$ So long term value of M is 35.36 grams	**M1** **A1** **M1** **A1ft** **[4]**	 $25\sqrt{2}$ or 35 or better soi ft their K.

2	**(i)**		h = 20, stops growing	B1 **[1]**	**AG** need interpretation	
	(ii)		$h = 20 - 20e^{-t/10}$ $dh/dt = 2e^{-t/10}$ $20e^{-t/10} = 20 - 20(1 - e^{-t/10}) = 20 - h$ $=10dh/dt$ when $t = 0$, $h = 20(1 - 1) = 0$ …………………………………………………… **OR** verifying by integration $\int \frac{dh}{20-h} = \int \frac{dt}{10}$ $\Rightarrow -\ln(20-h) = 0.1t + c$ $h = 0, t = 0, \Rightarrow c = -\ln 20$ $\Rightarrow \ln(20-h) = -0.1t + \ln 20$ $\Rightarrow \ln(\frac{20-h}{20}) = -0.1t$ $\Rightarrow 20 - h = 20e^{-0.1t}$ $\Rightarrow h = 20(1 - e^{-0.1t})$	M1A1 M1 A1 B1 M1 A1 B1 M1 A1 **[5]**	differentiation (for M1 need $ke^{-t/10}$, k const) oe eg $20 - h = 20 - 20(1 - e^{-t/10}) = 20e^{-t/10}$ $= 10dh/dt$ (showing sides equivalent) initial conditions ………………………………………………………………………….. sep correctly and intending to integrate correct result (condone omission of c, although no further marks are possible) condone ln (h – 20) as part of the solution at this stage constant found from expression of correct form (at any stage) but B0 if say $c = \ln(-20)$ (found using $\ln(h - 20)$) combining logs and anti-logging (correct rules) correct form (do not award if B0 above)	

Question			Answer	Marks	Guidance
2	(iii)		$\frac{200}{(20+h)(20-h)} = \frac{A}{20+h} + \frac{B}{20-h}$ $\Rightarrow$ 200 = $A(20 - h) + B(20 + h)$ $h = 20 \Rightarrow 200 = 40B$, $B = 5$ $h = -20 \Rightarrow 200 = 40A$, $A = 5$ 200 dh/dt = 400 – h^2 $\Rightarrow \int \frac{200}{400-h^2} dh = \int dt$ $\Rightarrow \int (\frac{5}{20+h} + \frac{5}{20-h}) dh = \int dt$ $\Rightarrow$ 5ln(20 + h) –5ln(20 – h) = $t + c$ When $t = 0$, $h = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$ $\Rightarrow 5\ln\frac{20+h}{20-h} = t$ $\Rightarrow \frac{20+h}{20-h} = e^{t/5}$ $\Rightarrow$ 20 + h = – h)$e^{t/5}$ = 20 $^{t/5}$ – h $e^{t/5}$ $\Rightarrow$ $h + h e^{t/5} = 20 e^{t/5} - 20$ $\Rightarrow$ $h(e^{t/5} + 1) = 20(e^{t/5} - 1)$ $\Rightarrow h = \frac{20(e^{t/5}-1)}{e^{t/5}+1}$ $\Rightarrow h = \frac{20(1-e^{-t/5})}{1+e^{-t/5}}$ *	M1 A1 A1 M1 A1 B1 M1 DM1 A1 **[9]**	cover up, substitution or equating coeffs separating variables and intending to integrate (condone sign error) substituting partial fractions ft their A,B, condone absence of c, Do not allow ln(h-20) for A1. cao need to **show** this. c can be found at any stage. **NB *c* = ln(–1)** (from ln(h –20)) or similar **scores B0.** anti-logging an equation of the correct form . Allow if $c = 0$ clearly stated (provided that $c = 0$) even if B mark is not awarded, but do not allow if c omitted. Can ft their c. maki h the subject, dependent on previous mark **NB** method marks can be in either order, in which case the dependence is the other way around.(In which case, 20 + h is divided by 20 – h first to isolate h). **AG must have obtained B1** (for c) in order to obtain final A1**.**
2	(iv)		As $t \to \infty$, $h \to 20$. So long-term height is 20m.	B1 **[1]**	www
2	(v)		1st model $h = 20(1 - e^{-0\ 1}) = 1.90$.. 2nd model $h = 20(e^{1/5} - 1)/(e^{1/5} + 1) = 1.99$.. so 2nd model fits data better	B1 B1 B1 dep **[3]**	O 1st model $h = 2$ gives $t = 1.05$.. 2nd model $h = 2$ gives $t = 1.003$.. dep previous B1s correct

Question			answer	Marks	Guidance
3	**(i)**		$dV/dx = \pi(20x - x^2)$ $\Rightarrow \quad \frac{dV}{dt} = \frac{dV}{dx}.\frac{dx}{dt}$ $= \pi x(20-x).\frac{dx}{dt} = k(20-x)$ $\Rightarrow \quad \pi x\frac{dx}{dt} = k$ *	B1 M1 A1 A1 **[4]**	 o **ag**
.3	**(ii)**		$\int \pi x\,dx = \int k\,dt$ $\Rightarrow \quad ½\ \pi x^2 = kt + c$ When $t = 0$, $x = 0 \Rightarrow c = 0$ $\Rightarrow \quad ½\ \pi x^2 = kt$ Full when $x = 10$, $t = T$ $\Rightarrow \quad 50\pi = kT$ $\Rightarrow \quad T = 50\pi/k$ *	M1 A1 B1 M1 A1 **[5]**	separate variables and attempt integration of both sides condone absence of c c=0 www substitut t or T=50 π/k or x=10 and rearranging for the other (dependent on first M1) oe **ag,** need to have c=0
3	**(iii)**		$dV/dt = -kx$ $\Rightarrow \ \pi x(20-x).\frac{dx}{dt} = -kx$ $\Rightarrow \ \pi(20-x)\frac{dx}{dt} = -k$*	B1 M1 A1 **[3]**	correct $dV/dx.dx/dt = \pm kx$ ft **ag**

Question			Answer	Marks	Guidance
3	**(iv)**		$\int \pi(20-x)\,\mathrm{d}x = \int -k\,\mathrm{d}t$	M1	separate variables and intend to integrate both sides
			$\pi(20x - ½x^2) = -kt + c$	B1 A1	LHS (not dependent on M1) RHS ie $-kt + c$ (condone absence of c)
			When $t = 0$, $x = 10$ $\Rightarrow$ $\pi(200 - 50) = c$ $\Rightarrow$ $c = 150\pi$	A1	evaluatio of c cao oe (x=10, t=0)
			$\Rightarrow$ $\pi(20x - ½x^2) = 150\pi - kt$ $x = 0$ when $150\pi - kt = 0$	M1	substitut x=0 and rearrange for t -dependent on first M1 and non-zero c ,oe
			$\Rightarrow$ $t = 150\pi/k = 3T$*	A1 **[6]**	**ag**

4(i) When $t = 0$, $v = 5(1 - e^0) = 0$ As $t \to \infty$, $e^{-2t} \to 0$, $\Rightarrow$ $v \to 5$ When $t = 0.5$, $v = 3.16$ m s^{-1}	E1 E1 B1 [3]	
(ii) $\frac{dv}{dt} = 5 \times (-2)e^{-2t} = 10e^{-2t}$ $10 - 2v = 10 - 10(1 - e^{-2t}) = 10e^{-2t}$ $\Rightarrow$ $\frac{dv}{dt} = 10 - 2v$	B1 M1 E1 [3]	
(iii) $\frac{dv}{dt} = 10 - 0.4v^2$ $\Rightarrow$ $\frac{10}{100 - 4v^2}\frac{dv}{dt} = 1$ $\Rightarrow$ $\frac{10}{25 - v^2}\frac{dv}{dt} = 4$ $\Rightarrow$ $\frac{10}{(5-v)(5+v)}\frac{dv}{dt} = 4$ * $\frac{10}{(5-v)(5+v)} = \frac{A}{5-v} + \frac{B}{5+v}$ $\Rightarrow$ $10 = A(5 + v) + B(5 - v)$ $v = 5 \Rightarrow 10 = 10A \Rightarrow A = 1$ $v = -5 \Rightarrow 10 = 10B \Rightarrow B = 1$ $\Rightarrow$ $\frac{10}{(5-v)(5+v)} = \frac{1}{5-v} + \frac{1}{5+v}$ $\Rightarrow$ $\int (\frac{1}{5-v} + \frac{1}{5+v})\,dv = 4\int dt$ $\Rightarrow$ $\ln(5+v) - \ln(5-v) = 4t + c$ when $t = 0$, $v = 0$, $\Rightarrow 0 = 4\times 0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln\left(\frac{5+v}{5-v}\right) = 4t$ $\Rightarrow$ $t = \frac{1}{4}\ln\left(\frac{5+v}{5-v}\right)$ *	M1 E1 M1 A1 M1 A1 A1 E1 [8]	for both A=1,B=1 separating variables correctly and indicating integration ft their A,B, condone absence of c ft finding c from an expression of correct form
(iv) When $t \to \infty$, $e^{-4t} \to 0$, $\Rightarrow$ $v \to 5/1 = 5$ when $t = 0.5$, $t = \frac{5(1-e^{-2})}{1+e^{-2}} = 3.8\,\text{m s}^{-1}$	E1 M1A1 [3]	
(v) The first model	E1 [1]	www