1 Solve the differential equation $\frac{dy}{dx} = \frac{y}{x(x+1)}$, given that when $x = 1$, $y = 1$. Your answer should express y explicitly in terms of x. **[8]**

2 Water is leaking from a container. After t seconds, the depth of water in the container is x cm, and the volume of water is V cm^3, where $V = \frac{1}{3}x^3$. The rate at which water is lost is proportional to x, so that $\frac{dV}{dt} = -kx$, where k is a constant.

(i) Show that $x\frac{dx}{dt} = -k$. **[3]**

Initially, the depth of water in the container is 10 cm.

(ii) Show by integration that $x = \sqrt{100 - 2kt}$. **[4]**

(iii) Given that the container empties after 50 seconds, find k. **[2]**

Once the container is empty, water is poured into it at a constant rate of 1 cm^3 per second. The container continues to lose water as before.

(iv) Show that, t seconds after starting to pour the water in, $\frac{dx}{dt} = \frac{1-x}{x^2}$. **[2]**

(v) Show that $\frac{1}{1-x} - x - 1 = \frac{x^2}{1-x}$.

Hence solve the differential equation in part **(iv)** to show that

$$t = \ln\left(\frac{1}{1-x}\right) - \frac{1}{2}x^2 - x.$$ **[6]**

(vi) Show that the depth cannot reach 1 cm. **[1]**

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3 A curve satisfies the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2y$, and passes through the point (1, 1). Find y in terms of x. **[4]**

4 A skydiver drops from a helicopter. Before she opens her parachute, her speed $v\,\mathrm{m\,s^{-1}}$ after time t seconds is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10\mathrm{e}^{-\frac{1}{2}t}.$$

When $t = 0$, $v = 0$.

(i) Find v in terms of t. [4]

(ii) According to this model, what is the speed of the skydiver in the long term? [2]

She opens her parachute when her speed is $10\,\mathrm{m\,s^{-1}}$. Her speed t seconds after this is $w\,\mathrm{m\,s^{-1}}$, and is modelled by the differential equation

$$\frac{\mathrm{d}w}{\mathrm{d}t} = -\frac{1}{2}(w-4)(w+5).$$

(iii) Express $\frac{1}{(w-4)(w+5)}$ in partial fractions. [4]

(iv) Using this result, show that $\frac{w-4}{w+5} = 0.4\mathrm{e}^{-4.5t}$. [6]

(v) According to this model, what is the speed of the skydiver in the long term? [2]

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5 Data suggest that the number of cases of infection from a particular disease tends to oscillate between two values over a period of approximately 6 months.

(a) Suppose that the number of cases, P thousand, after time t months is modelled by the equation $P = \dfrac{2}{2 - \sin t}$. Thus, when $t = 0, P = 1$.

(i) By considering the greatest and least values of $\sin t$, write down the greatest and least values of P predicted by this model. [2]

(ii) Verify that P satisfies the differential equation $\dfrac{dP}{dt} = \frac{1}{2} P^2 \cos t$. [5]

(b) An alternative model is proposed, with differential equation

$$\frac{dP}{dt} = \tfrac{1}{2}(2P^2 - P)\cos t. \qquad (*)$$

As before, $P = 1$ when $t = 0$.

(i) Express $\dfrac{1}{P(2P-1)}$ in partial fractions. [4]

(ii) Solve the differential equation (*) to show that

$$\ln\left(\frac{2P \quad 1}{P}\right) = \tfrac{1}{2}\sin t. \qquad [5]$$

This equation can be rearranged to give $P = \dfrac{1}{2 \quad e^{\frac{1}{2}\sin t}}$.

(iii) Find the greatest and least values of P predicted by this model. [4]

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6 **(a)** The number of bacteria in a colony is increasing at a rate that is proportional to the square root of the number of bacteria present. Form a differential equation relating x, the number of bacteria, to the time t. [2]

(b) In another colony, the number of bacteria, y, after time t minutes is modelled by the differential equation

$$\frac{dy}{dt} = \frac{10000}{\sqrt{y}}.$$

Find y in terms of t, given that $y = 900$ when $t = 0$. Hence find the number of bacteria after 10 minutes. [6]

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