Question			answer	Marks	Guidance
1			$\frac{dy}{dx} = \frac{y}{x(x+1)}$		
			$\Rightarrow \int \frac{1}{y} dy = \int \frac{1}{x(x+1)} dx$	M1	correctly separating variables and intending to integrate (ie need to see attempt at integration or integral signs)
			$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$	M1	partial fractions soi
			$\Rightarrow 1 = A(x+1) + Bx$		
			$x = 0 \Rightarrow A = 1$	A1	$A = 1$ www
			$x = -1 \Rightarrow 1 = -B \Rightarrow B = -1$	A1	$B = -1$ www
			$\Rightarrow \ln y = \int (\frac{1}{x} - \frac{1}{x+1}) dx = \ln x - \ln(x+1) + c$	A1	ft their A,B condone absence of c or ln$c$
			$x = 1, y = 1 \Rightarrow 0 = 0 - \ln 2 + c \Rightarrow c = \ln 2$	B1	evaluating their c **at any stage** dependent on x and y terms all being logs of correct form but **do not award following incorrect log rules,** ft their A,B**.** c could be say a decimal. (eg $y = x/(x+1) + c$ then c being found is B0)
			$\Rightarrow \ln y = \ln x - \ln(x+1) + \ln 2 = \ln(2x/(x+1))$	M1	correctly combining lns and antilogging throughout (must have included the constant term). Apply this strictly. Do not allow if c is included as an afterthought unless completely convinced. ft A,B Logs must be of correct form ie not following say $\int \frac{1}{x(x+1)} dx = \ln(x^2 + x)$ unless ft from partial fractions and $B = 1$
			$\Rightarrow y = 2x/(x+1)$	A1	cao www $(y = e^{693}\left(\frac{x}{x+1}\right)$ loses final A1) **NB** evaluating c and log work can be in either order. eg $y = cx/(x+1)$, at $x = 1$, $y = 1$, $c = 2$
				[8]	

2(i) $\frac{dV}{dt} = -kx$ $V = 1/3\, x^3 \Rightarrow dV/dx = x^2$ $\frac{dV}{dt} = \frac{dV}{dx}.\frac{dx}{dt} = x^2 \frac{dx}{dt}$ $\Rightarrow x^2 \frac{dx}{dt} = -kx$ $\Rightarrow x\frac{dx}{dt} = -k$ *	B1 M1 A1 [3]	oe eg $dx/dt = dx/dV \,.\, dV/dt = 1/x^2 . -kx = -k/x$ **AG**	
(ii) $x\frac{dx}{dt} = -k \Rightarrow \int x\,dx = \int -k\,dt$ $\Rightarrow \frac{1}{2}x^2 = -kt + c$ When $t = 0$, $x = 10 \Rightarrow 50 = c$ $\Rightarrow \frac{1}{2}x^2 = 50 - kt$ $\Rightarrow x = \sqrt{(100 - 2kt)}$ *	M1 A1 B1 A1 [4]	separating variables and intention to integrate condone absence of c finding c correctly ft their integral of form $ax^2 = bt + c$ where a,b non zero constants **AG**	
(iii) When $t = 50$, $x = 0$ $\Rightarrow 0 = 100 - 100k \Rightarrow k = 1$	M1 A1 [2]		
(iv) $dV/dt = 1 - kx = 1 - x$ $\Rightarrow x^2 dx/dt = 1 - x$ $\Rightarrow \frac{dx}{dt} = \frac{1-x}{x^2}$ *	M1 A1 [2]	for $dV/dt = 1 - kx$ or better **AG**	
(v) $\frac{1}{1-x} - x - 1 = \frac{1-(1-x)x-(1-x)}{1-x}$ $= \frac{1-x+x^2-1+x}{1-x} = \frac{x^2}{1-x}$ * $\int \frac{x^2}{1-x}dx = \int dt \Rightarrow \int (\frac{1}{1-x} - x - 1)dx = t + c$ $\Rightarrow -\ln(1-x) - \frac{1}{2}x^2 - x = t + c$ When $t = 0$, $x = 0 \Rightarrow c = -\ln 1 - 0 - 0 = 0$ $\Rightarrow t = \ln\left(\frac{1}{1-x}\right) - \frac{1}{2}x^2 - x$ *	M1 A1 M1 A1 B1 A1 [6]	combining to single fraction **AG** separating variables & subst for $x^2/(1-x)$ and intending to integrate condone absence of c finding c for equation of correct form eg $c = 0$, or $\pm \ln 1$ (allow $c=0$ without evaluation here) cao **AG**	or long division or cross multiplying check signs need both sides of integral accept $\ln(1/(1-x))$ as $-\ln(1-x)$ www ie $a\ln(1-x) + bx^2 + dx = et + c$ a,b,d,e non zero constants do not allow if c=0 without evaluation
(vi) understanding that ln (1/0) or 1/0 is undefined oe	B1 [1]	www	$\ln(1/0) = \ln 0$, $1/0 = \infty$ and $\ln(1/0) = \infty$ are all B0

3 $\frac{dy}{dx} = 3x^2 y$		
$\Rightarrow \int \frac{dy}{y} = \int 3x^2\, dx$	M1	separating variables
$\Rightarrow \ln y = x^3 + c$	A1	condone absence of c
When $x = 1$, $y = 1$, $\Rightarrow \ln 1 = 1 + c \Rightarrow c = -1$	B1	$c = -1$ oe
$\Rightarrow \ln y = x^3 - 1$		
$\Rightarrow y = e^{x^3 - 1}$	A1 [4]	o.e.

4(i) $v = \int 10e^{-\frac{1}{2}t}dt$ $= -20e^{-\frac{1}{2}t} + c$ when $t = 0$, $v = 0$ $\Rightarrow 0 = -20 + c$ $\Rightarrow c = 20$ so $v = 20 - 20e^{-\frac{1}{2}t}$	M1 A1 M1 A1 [4]	separate variables and intend to integrate $-20e^{-\frac{1}{2}t}$ finding c cao
(ii) As $t \to \infty$ $e^{-1/2\,t} \to 0$ $\Rightarrow v \to 20$ So long term speed is 20 m s^{-1}	M1 A1 [2]	 ft (for their $c>0$, found)
(iii) $\frac{1}{(w-4)(w+5)} = \frac{A}{w-4} + \frac{B}{w+5}$ $= \frac{A(w+5)+B(w-4)}{(w-4)(w+5)}$ $\Rightarrow 1 \equiv A(w+5) + B(w-4)$ $w = 4$: $1 = 9A \Rightarrow A = 1/9$ $w = -5$: $1 = -9B \Rightarrow B = -1/9$ $\Rightarrow \frac{1}{(w-4)(w+5)} = \frac{1/9}{w-4} - \frac{1/9}{w+5}$ $= \frac{1}{9(w-4)} - \frac{1}{9(w+5)}$	M1 M1 A1 A1 [4]	 cover up, substitution or equating coeffs 1/9 −1/9
(iv) $\frac{dw}{dt} = -\frac{1}{2}(w-4)(w+5)$ $\Rightarrow \int \frac{dw}{(w-4)(w+5)} = \int -\frac{1}{2}dt$ $\Rightarrow \int [\frac{1}{9(w-4)} - \frac{1}{9(w+5)}]dw = \int -\frac{1}{2}dt$ $\Rightarrow \frac{1}{9}\ln(w-4) - \frac{1}{9}\ln(w+5) = -\frac{1}{2}t + c$ $\Rightarrow \frac{1}{9}\ln\frac{w-4}{w+5} = -\frac{1}{2}t + c$ When $t = 0$, $w = 10 \Rightarrow c = \frac{1}{9}\ln\frac{6}{15} = \frac{1}{9}\ln\frac{2}{5}$ $\Rightarrow \ln\frac{w-4}{w+5} = -\frac{9}{2}t + \ln\frac{2}{5}$ $\Rightarrow \frac{w-4}{w+5} = e^{-\frac{9}{2}t+\ln\frac{2}{5}} = \frac{2}{5}e^{-\frac{9}{2}t} = 0.4e^{-4.5t}$ *	M1 M1 A1ft M1 M1 E1 [6]	separating variables substituting their partial fractions integrating correctly (condone absence of c) correctly evaluating c (at any stage) combining lns (at any stage) www
(v) As $t \to \infty$ $e^{-4.5t} \to 0$ $\Rightarrow w - 4 \to 0$ So long term speed is 4 m s^{-1}.	M1 A1 [2]	

5 (a) (i) $P_{max} = \frac{2}{2-1} = 2$ $P_{min} = \frac{2}{2+1} = 2/3.$	B1 B1 [2]	
(ii) $P = \frac{2}{2-\sin t} = 2(2-\sin t)^{-1}$ $\Rightarrow \quad \frac{dP}{dt} = -2(2-\sin t)^{-2}.-\cos t$ $= \frac{2\cos t}{(2-\sin t)^2}$ $\frac{1}{2}P^2\cos t = \frac{1}{2}\frac{4}{(2-\sin t)^2}\cos t$ $= \frac{2\cos t}{(2-\sin t)^2} = \frac{dP}{dt}$	M1 B1 A1 DM1 E1 [5]	chain rule $-1(\ldots)^{-2}$ soi (or quotient rule M1,numerator A1,denominator A1) attempt to verify or by integration as in (b)(ii)
(b)(i) $\frac{1}{P(2P-1)} = \frac{A}{P} + \frac{B}{2P-1}$ $= \frac{A(2P-1)+BP}{P(2P-1)}$ $\Rightarrow \quad 1 = A(2P-1) + BP$ $P = 0 \Rightarrow 1 = -A \Rightarrow A = -1$ $P = ½ \Rightarrow 1 = A.0 + ½ B \Rightarrow B = 2$ So $\frac{1}{P(2P-1)} = -\frac{1}{P} + \frac{2}{2P-1}$	M1 M1 A1 A1 [4]	correct partial fractions substituting values, equating coeffs or cover up rule $A = -1$ $B = 2$
(ii) $\frac{dP}{dt} = \frac{1}{2}(2P-P^2)\cos t$ $\Rightarrow \quad \int \frac{1}{2P^2-P}dP = \int \frac{1}{2}\cos t dt$ $\Rightarrow \quad \int (\frac{2}{2P-1} - \frac{1}{P})dP = \int \frac{1}{2}\cos t dt$ $\Rightarrow \quad \ln(2P-1) - \ln P = ½ \sin t + c$ When $t = 0$, $P = 1$ $\Rightarrow \quad \ln 1 - \ln 1 = ½ \sin 0 + c \Rightarrow c = 0$ $\Rightarrow \ln(\frac{2P-1}{P}) = \frac{1}{2}\sin t$ *	M1 A1 A1 B1 E1 [5]	separating variables $\ln(2P-1) - \ln P$ ft their A,B from (i) ½ sin t finding constant = 0
(iii) $P_{max} = \frac{1}{2-e^{1/2}} = 2.847$ $P_{min} = \frac{1}{2-e^{-1/2}} = 0.718$	M1A1 M1A1 [4]	www www

6 (a) $\frac{dx}{dt} = k\sqrt{x}$	M1 A1 [2]	$\frac{dx}{dt} = ...$ $k\sqrt{x}$
(b) $\frac{dy}{dt} = \frac{10000}{\sqrt{y}}$ $\Rightarrow \int \sqrt{y}dy = \int 10000dt$ $\Rightarrow \frac{2}{3}y^{\frac{3}{2}} = 10000t + c$ When $t = 0$, $y = 900 \Rightarrow 18000 = c$ $\Rightarrow y = [\frac{3}{2}(10000t + 18000)]^{\frac{2}{3}}$ $= (1500(10t+18))^{\frac{2}{3}}$ When $t = 10$, $y = 3152$	M1 A1 B1 A1 M1 A1 [6]	separating variables condone omission of c evaluating constant for their integral any correct expression for y = for method allow substituting t=10 in their expression cao