Question			Answer	Marks	Guidance
1	(i)		$\frac{1}{(1+2x)(1-x)} = \frac{A}{1+2x} + \frac{B}{1-x} \Rightarrow 1 = A(1-x) + B(1+2x)$		
				M1	Cover up, substitution or equating coefficients
			$x = 1 \Rightarrow 3B = 1$, $B = 1/3$	A1	
			$x = -½ \Rightarrow 1 = 3A/2$, $A = 2/3$	A1	isw after correct A and B stated
				[3]	
1	(ii)		$1 + x - 2x^2 = (1+2x)(1-x)$	B1	May be seen in separation of variables (may be implied by later working) – implied by the use of factors $(1 + 2x)$ and $(1 - x)$
			$\Rightarrow \quad \frac{1}{3}\int[\frac{2}{(1+2x)} + \frac{1}{1-x}]\mathrm{d}x = \int k\mathrm{d}t$	M1	Separating variables and substituting partial fractions. If no subsequent work integral signs needed, but allow omission of $\mathrm{d}x$ or $\mathrm{d}t$, but must be correctly placed if present
			$\lambda \ln(1+2x) + \mu \ln(1-x) = kt(+c)$	A1	Any non-zero constant λ, μ
			$\Rightarrow \quad \ln(1 + 2x) - \ln(1 - x) = 3kt\ (+ c)$	A1	www oe (condone absence of c)
			When $t = 0, x = 0 \Rightarrow c = 0$	B1	cao (**must** follow previous A1) need to **show** (at some stage) that $c = 0$. s a minimum $t = 0, x = 0, c = 0$. Note that $c = \ln(-1)$ (usually from incorrect integration of $(1 - x)$) or similar scores B0
			$\Rightarrow \quad \ln\left(\frac{1+2x}{1-x}\right) = 3kt$	M1	Combining both their log terms correctly. Follow through their c. Allow if $c = 0$ clearly stated (provided that $c = 0$) even if B mark is not awarded, but do not allow if c omitted
			$\Rightarrow \quad \frac{1+2x}{1-x} = e^{3kt}$ *	A1	**AG** www must have obtained all previous marks in this part
				[7]	

Question			Answer	Marks	Guidance
1	(iii)		$(1 + 2(0.75)) / (1 – 0.75)=e^{3k}$	M1	substituting $t = 1$, $x = 0.75$ at any stage
			$k = (1/3)\ln 10$ (= 0.768 (3 s.f.))	A1	3sf or better
			$t = \ln(2.8/0.1)/3k = 1.45$ hours	A1	1.45 (or better) or 1 hr 27 mins
				[3]	
1	(iv)		$1 + 2x = e^{3kt} – xe^{3kt}$		
			$\Rightarrow\quad 2x + xe^{3kt} = e^{3kt} – 1$	M1*	Multiplying out and collecting x terms (condone one error)
			$\Rightarrow\quad x(2 + e^{3kt}) = e^{3kt} – 1$	M1dep*	Factorising their x terms correctly
			$\Rightarrow\quad x = (e^{3kt} – 1)/ (2 + e^{3kt})$	A1	
			$= (1 – e^{-3kt})/(1 + 2e^{-3kt})$ *	A1	www (**AG**) – as AG must be an indication of how previous line leads to the required result (eg stating or showing multiplying by e^{-3kt})
			when $t \to \infty\quad e^{-3kt} \to 0$ $x = (1 – e^{-3kt})/(1 + 2e^{-3kt}) \to 1/1 = 1$	B1	clear indication that $e^{-3kt} \to 0$ so, for example, accept as a minimum $(x \to)\dfrac{1-0}{1+0} = 1$ **or** $e^{-3kt} \to 0 \Rightarrow (x \to)\ 1$ (NB substitution of large values of t with no further explanation is B0)
				[5]	
		OR	$\dfrac{1-x}{1+2x} = e^{-3kt}$	B1	
			$1 - x = e^{-3kt} + 2xe^{-3kt}$	M1*	Multiplying up and expanding (condone one error)
			$x(1 + 2e^{-3kt}) = 1 - e^{-3kt}$	M1dep*	Factorising their x terms correctly
			$x = (1 – e^{-3kt})/(1 + 2e^{-3kt})$ *	A1	www (**AG**) – final B mark as in scheme above

Question			Answer	Marks	Guidance
2	**(i)**		EITHER		
			$x = e^{3t}, y = te^{2t}$	B1	soi
			$dy/dt = 2te^{2t} + e^{2t}$	M1	Their $dy/dt \div dx/dt$ in terms of t
			$\Rightarrow$ $dy/dx = (2te^{2t} + e^{2t})/3e^{3t}$	A1	oe cao allow for unsimplified form even if subsequently cancelled incorrectly ie can isw
			when $t = 1$, $dy/dx = 3e^2/3e^3 = 1/e$	A1	cao www must be simplified to 1/e oe
			OR		
			$3t = \ln x,\ y = \dfrac{\ln x}{3} e^{2/3 \ln x} = \dfrac{x^{2/3}\ln x}{3}$	B1	Any equivalent form of y in terms of x only
			$dy/dx = \dfrac{1}{3}x^{2/3}\dfrac{1}{x} + \ln x \dfrac{2}{9}x^{-1/3}$	M1	Differentiating their y provided not eased ie need a product including $\ln kx$ and x^p and subst $x = e^{3t}$ to obtain dy/dx in terms of t
			$= \dfrac{1}{3e^t} + \dfrac{2t}{3e^t}$	A1	oe cao
			$dy/dx = 1/3e + 2/3e = 1/e$	A1	www cao **exact only** must be simplified to 1/e or e^{-1}
				[4]	
2	**(ii)**		$3t = \ln x \Rightarrow t = (\ln x)/3$	B1	Finding t correctly in terms of x
			$y = (\ln x)/3e^{(2\ln x)/3}$	M1	Subst in y using their t
			$y = \dfrac{1}{3}x^{\frac{2}{3}}\ln x$	A1	Required form $ax^b \ln x$ only
					NB If this work was already done in 5(i), marks can only be scored in 5(ii) if candidate specifically refers in this part to their part (i).
				[3]	

Question			Answer	Marks	Guidance
3	(i)		Either $h = (1 - ½ At)^2 \Rightarrow dh/dt = -A(1 - ½ At)$	M1	Including function of a function, need to see middle step
			$= -A\sqrt{h}$	A1	AG
			when $t = 0$, $h = (1 - 0)^2 = 1$ as required	B1	
			OR		
			$\int \frac{dh}{\sqrt{h}} = \int -A\,dt$	M1	Separating variables correctly and integrating
			$2h^{1/2} = -At + c$	A1	Including c. [Condone change of c.]
			$h = \left(\frac{-At + c}{2}\right)^2$ at $t = 0$, $h = 1$, $1 = (c/2)^2 \Rightarrow c = 2$, $h = (1 - At/2)^2$	B1	Using initial conditions AG
				[3]	
3	(ii)		When $t = 20$, $h = 0$	M1	Subst and solve for A
			$\Rightarrow 1 - 10A = 0$, $A = 0.1$	A1	cao
			When the depth is 0.5 m, $0.5 = (1 - 0.05t)^2$	M1	substitute $h = 0.5$ and their A and solve for t
			$\Rightarrow \quad 1 - 0.05t = \sqrt{0.5}$, $t = (1 - \sqrt{0.5})/0.05 = 5.86$s	A1	www cao accept 5.9
				[4]	

Question			Answer	Marks	Guidance
3	(iii)		$\frac{dh}{dt} = -B\frac{\sqrt{h}}{(1+h)^2}$ $\Rightarrow \int \frac{(1+h)^2}{\sqrt{h}}\,dh = -\int B\,dt$	M1	separating variables correctly and intend to integrate **both sides** (may appear later) [**NB reading (1+*h*)²as 1+*h*² eases the question. Do not mark as a MR]** In cases where (1+*h*)² is MR as 1+*h*²or incorrectly expanded, as say 1+*h*+*h*² or 1+*h*², allow first M1 for correct separation and attempt to integrate and can then score a max of M1M0A0A0A1 (for –*Bt*+*c*) A0A0, max 2/7.
			EITHER, LHS		
			$\int \frac{1+2h+h^2}{\sqrt{h}}\,dh$ $= \int (h^{-1/2} + 2h^{1/2} + h^{3/2})\,dh$	M1 A1	expanding $(1+h)^2$ and dividing by $\sqrt{h}$ to form a one line function of *h* (indep of first M1) with each term expressed as a single power of *h* eg must simplify say $1/\sqrt{h}+2h/\sqrt{h}+h^2\sqrt{h}$,condone a single error for M1 (do not need to see integral signs) $h^{-1/2} + 2h^{1/2} + h^{3/2}$ cao dep on second M only -do not need integral signs
			OR ,LHS, **EITHER**		
			$(1+2h+h^2)2h^{1/2} - \int 2h^{1/2}(2+2h)\,dh$ **OR** $h^{1/2} + h^{3/2} + \frac{h^{5/2}}{3} + \int \frac{1}{2}h^{-3/2}(h+h^2+\frac{h^3}{3})\,dh$	M1 A1	using $\int u\,dv = uv - \int v\,du$ correct formula used correctly, indep of first M1 condone a single error for M1if intention clear cao oe
			$2h^{1/2} + \frac{4h^{3/2}}{3} + \frac{2h^{5/2}}{5}$	A1	cao **oe**, **both sides dependent on first M1 mark**
			$= -Bt + c$	A1	cao need –*Bt* and *c* for second A1 but the constant may be on either side
			$\Rightarrow 2h^{1/2} + 4h^{3/2}/3 + 2h^{5/2}/5 = -Bt + c$		
			When $t = 0$, $h = 1 \Rightarrow c = 56/15$	A1	from correct work only (accept 3.73 or rounded answers here but not for final A1) or *c*= –56/15 if constant on opposite side.
			$\Rightarrow h^{1/2}(30 + 20h + 6h^2) = 56 - 15Bt$ *	A1	NB **AG must be from all correct exact work including exact *c*.**
				[7]	

Question			Answer	Marks	Guidance
3	**(iv)**		$h = 0$ when $t = 20$	M1	Substituting $h = 0, t = 20$
			$\Rightarrow B = 56/300 = 0.187$	A1	Accept 0.187
			When $h = 0.5$ $56 - 2.8t = 29.3449\ldots$	M1	Subst their $h = 0.5$, ft their B and attempt to solve
			$\Rightarrow t = 9.52$s	A1	Accept answers that round to 9.5s www.
				[4]	

Question			Answer	Marks	Guidance
4	(i)		$v\,dv/dx + 4x = 0$		
			$\int v\,dv = -\int 4x\,dx$	M1	separating variables and intending to integrate
			$½\,v^2 = -2x^2 + c$	A1	oe condone absence of c. [Not immediate $v^2 = -4x^2$ (+c)]
			When $x = 1$, $v = 4$, so $c = 10$	B1	finding c, must be convinced as AG, need to see at least the statement given here oe (condone change of c)
			so $v^2 = 20 - 4x^2$ *	A1	**AG** following finding c convincingly Alternatively, **SC** $v^2=20-4x^2$, by differentiation, $2v\,dv/dx= -8x$ $v\,dv/dx + 4x =0$ scores B2 if , in addition, they check the initial conditions a further B1 is scored (ie 16=20-4). Total possible 3/4.
				[4]	
4	(ii)		$x = \cos 2t + 2\sin 2t$		
			when $t = 0$, $x = \cos 0 + 2\sin 0 = 1$*	B1	**AG** need some justification
			$v = dx/dt = -2\sin 2t + 4\cos 2t$	M1	differentiating, accept ±2,±4 as coefficients but not ±1,±2 and not ±1/2,±1 from integrating
				A1	cao
			$v = 4\cos 0 - 2\sin 0 = 4$*	A1	ww AG
				[4]	

Question			Answer	Marks	Guidance
4	(iii)		$\cos 2t + 2\sin 2t = R\cos(2t - \alpha) = R(\cos 2t\cos\alpha + \sin 2t\sin\alpha)$		**SEE APPENDIX 1 for further guidance**
			$R = \sqrt{5}$	B1	or 2.24 or better (not ± unless negative rejected)
			$R\cos\alpha = 1$, $R\sin\alpha = 2$	M1	correct pairs soi
			$\tan\alpha = 2$,	M1	correct method
			$\alpha = 1.107$	A1	cao radians only, 1.11 or better (or multiples of π that round to 1.11)
			$x = \sqrt{5}\cos(2t - 1.107)$ $v = -2\sqrt{5}\sin(2t - 1.107)$	A1	differentiating or otherwise, ft their numerical R, α (not degrees) required form **SC** B1 for $v = \sqrt{20}\cos(2t + 0.464)$ oe
			EITHER $v^2 = 20\sin^2(2t - \alpha)$ $20 - 4x^2 = 20 - 20\cos^2(2t - \alpha)$	M1	squarin their v (if of required form with same α as x), and x, and attempting to show $v^2 = 20 - 4x^2$ ft their R, α (incl. degrees) [α may not be specified].
			$= 20(1 - \cos^2(2t - \alpha)) = 20\sin^2(2t - \alpha)$ so $v^2 = 20 - 4x^2$	A1	cao www (condone the use of over-rounded α (radians) or degrees)
			OR multiplying out $v^2 = (-2\sin 2t + 4\cos 2t)^2$ $= 4\sin^2 2t - 16\sin 2t\cos 2t + 16\cos^2 2t$ and $4x^2 = 4(\cos^2 2t + 4\sin 2t\cos 2t + 4\sin^2 2t)$ $= 4\cos^2 2t + 16\sin 2t\cos 2t + 16\sin^2 2t$ (need middle term) and attempting to show that $v^2 + 4x^2 = 4(\sin^2 2t + \cos^2 2t) + 16(\cos^2 2t + \sin^2 2t)$ $= 4 + 16 = 20$ (or $20 - 4x^2 = v^2$) oe	M1	differentiating to find v (condone coefficient errors), squaring v and x and multiplying out (need attempt at middle terms) and attempting to show $v^2 = 20 - 4x^2$
			so $v^2 = 20 - 4x^2$	A1	cao www
				[7]	

Question			Answer	Marks	Guidance
4	(iv)		$x = \sqrt{5}\cos(2t - \alpha)$ or otherwise x max = $\sqrt{5}$ when $\cos(2t - \alpha) = 1$, $2t - 1.107 = 0$, $2t = 1.107$ $t = 0.55$	B1 M1 A1 **[3]**	ft their R oe (say by differentiation) ft their α in radians or degrees for method only cao (or answers that round to 0.554)

Question			answer	Marks	Guidance
5	(i)		dV/dt = $k\sqrt{V}$	B1	cao condone different k (allow MR B1 for = kV^2)
			$V = (½\, kt + c)^2$ ⇒ dV/dt = 2(½ kt + c).½ k	M1	2(1/2 kt + c) × constant multiple of k (or from multiplying out oe; or implicit differentiation)
			= k(½ kt + c)	A1	cao www any equivalent form (including unsimplified)
			= $k\sqrt{V}$	A1	Allow SCB2 if V=(1/2 kt + c)² fully obtained by integration including convincing change of constant if used Can score B1 M0 SCB2
				[4]	
	(ii)		$(½\, k + c)^2$ = 10 000 ⇒ ½ k + c = 100	B1	substituting any one from t = 1, V = 10,000 or t = 0, V = 0 or t = 2, V = 40,000 into squared form or rooted form of equation (Allow −/±100 or −/±200)
			$(k + c)^2$ = 40 000 ⇒ k + c = 200 ⇒ ½ k = 100	B1	substituting any other from above
			⇒ k = 200, c = 0	M1	Solving correct equations for both www (possible solutions are (200,0), (−200,0), (600, −400), (−600,400) (some from –ve root))
			⇒ $V = (100t)^2 = 10000t^2$	A1	either form www SC B2 for V = (100t)² oe stated without justification SCB4 if justification eg showing substitution SC those working with (k + c)² = 30,000 can score a maximum of B1B0 M1A0 (leads to k ≈ 146, c ≈ 26.8)
				[4]	