

1 Solve the equation $\frac{5x}{2x+1} - \frac{3}{x+1} = 1$. [5]

2 Express $\frac{3x}{(2-x)(4+x^2)}$ in partial fractions. [5]

3 Solve the equation $\frac{4x}{x+1} - \frac{3}{2x+1} = 1$. [5]

4 Express $\frac{1}{(2x+1)(x^2+1)}$ in partial fractions. [5]

5 Express $\frac{x}{x^2-1} + \frac{2}{x+1}$ as a single fraction, simplifying your answer. [3]

- 6 Find the first three terms in the binomial expansion of $\sqrt{4+x}$ in ascending powers of x .
State the set of values of x for which the expansion is valid. [5]

- 7 (i) Express $\frac{3}{(y-2)(y+1)}$ in partial fractions. [3]

(ii) Hence, given that x and y satisfy the differential equation

$$\frac{dy}{dx} = x^2(y-2)(y+1),$$

show that $\frac{y-2}{y+1} = Ae^{x^3}$, where A is a constant. [5]

- 8 Express $\frac{x}{x^2-4} + \frac{2}{x+2}$ as a single fraction, simplifying your answer. [3]

- 9 (i) Find the first three non-zero terms of the binomial series expansion of $\frac{1}{\sqrt{1+4x^2}}$, and state the set of values of x for which the expansion is valid. [5]

(ii) Hence find the first three non-zero terms of the series expansion of $\frac{1-x^2}{\sqrt{1+4x^2}}$. [3]

10 Two students are trying to evaluate the integral $\int_1^2 \sqrt{1+e^{-x}} dx$.

Sarah uses the trapezium rule with 2 strips, and starts by constructing the following table.

| | | | |
|-------------------|--------|--------|--------|
| x | 1 | 1.5 | 2 |
| $\sqrt{1+e^{-x}}$ | 1.1696 | 1.1060 | 1.0655 |

(i) Complete the calculation, giving your answer to 3 significant figures. [2]

Anish uses a binomial approximation for $\sqrt{1+e^{-x}}$ and then integrates this.

(ii) Show that, provided e^{-x} is suitably small, $(1+e^{-x})^{\frac{1}{2}} \approx 1 + \frac{1}{2}e^{-x} - \frac{1}{8}e^{-2x}$. [3]

(iii) Use this result to evaluate $\int_1^2 \sqrt{1+e^{-x}} dx$ approximately, giving your answer to 3 significant figures. [3]