1 (a) (i) $P_{max} = \frac{2}{2-1} = 2$ $P_{min} = \frac{2}{2+1} = 2/3.$	B1 B1 [2]	
(ii) $P = \frac{2}{2-\sin t} = 2(2-\sin t)^{-1}$ $\Rightarrow \quad \frac{dP}{dt} = -2(2-\sin t)^{-2}.-\cos t$ $= \frac{2\cos t}{(2-\sin t)^2}$ $\frac{1}{2}P^2\cos t = \frac{1}{2}\frac{4}{(2-\sin t)^2}\cos t$ $= \frac{2\cos t}{(2-\sin t)^2} = \frac{dP}{dt}$	M1 B1 A1 DM1 E1 [5]	chain rule $-1(\ldots)^{-2}$ soi (or quotient rule M1,numerator A1,denominator A1) attempt to verify or by integration as in (b)(ii)
(b)(i) $\frac{1}{P(2P-1)} = \frac{A}{P} + \frac{B}{2P-1}$ $= \frac{A(2P-1)+BP}{P(2P-1)}$ $\Rightarrow \quad 1 = A(2P-1) + BP$ $P = 0 \Rightarrow 1 = -A \Rightarrow A = -1$ $P = ½ \Rightarrow 1 = A.0 + ½ B \Rightarrow B = 2$ So $\frac{1}{P(2P-1)} = -\frac{1}{P} + \frac{2}{2P-1}$	M1 M1 A1 A1 [4]	correct partial fractions substituting values, equating coeffs or cover up rule $A = -1$ $B = 2$
(ii) $\frac{dP}{dt} = \frac{1}{2}(2P-P^2)\cos t$ $\Rightarrow \quad \int \frac{1}{2P^2-P}dP = \int \frac{1}{2}\cos t dt$ $\Rightarrow \quad \int (\frac{2}{2P-1} - \frac{1}{P})dP = \int \frac{1}{2}\cos t dt$ $\Rightarrow \quad \ln(2P-1) - \ln P = ½ \sin t + c$ When $t = 0$, $P = 1$ $\Rightarrow \quad \ln 1 - \ln 1 = ½ \sin 0 + c \Rightarrow c = 0$ $\Rightarrow \ln(\frac{2P-1}{P}) = \frac{1}{2}\sin t$ *	M1 A1 A1 B1 E1 [5]	separating variables $\ln(2P-1) - \ln P$ ft their A,B from (i) ½ $\sin t$ finding constant = 0
(iii) $P_{max} = \frac{1}{2-e^{1/2}} = 2.847$ $P_{min} = \frac{1}{2-e^{-1/2}} = 0.718$	M1A1 M1A1 [4]	www www

2(i) When $t = 0$, $v = 5(1 - e^{0}) = 0$ As $t \to \infty$, $e^{-2t} \to 0$, $\Rightarrow$ $v \to 5$ When $t = 0.5$, $v = 3.16 \text{ m s}^{-1}$	E1 E1 B1 [3]	
(ii) $\frac{dv}{dt} = 5 \times (-2)e^{-2t} = 10e^{-2t}$ $10 - 2v = 10 - 10(1 - e^{-2t}) = 10e^{-2t}$ $\Rightarrow$ $\frac{dv}{dt} = 10 - 2v$	B1 M1 E1 [3]	
(iii) $\frac{dv}{dt} = 10 - 0.4v^2$ $\Rightarrow$ $\frac{10}{100 - 4v^2}\frac{dv}{dt} = 1$ $\Rightarrow$ $\frac{10}{25 - v^2}\frac{dv}{dt} = 4$ $\Rightarrow$ $\frac{10}{(5-v)(5+v)}\frac{dv}{dt} = 4$ * $\frac{10}{(5-v)(5+v)} = \frac{A}{5-v} + \frac{B}{5+v}$ $\Rightarrow$ $10 = A(5 + v) + B(5 - v)$ $v = 5 \Rightarrow 10 = 10A \Rightarrow A = 1$ $v = -5 \Rightarrow 10 = 10B \Rightarrow B = 1$ $\Rightarrow$ $\frac{10}{(5-v)(5+v)} = \frac{1}{5-v} + \frac{1}{5+v}$ $\Rightarrow$ $\int (\frac{1}{5-v} + \frac{1}{5+v})\,dv = 4\int dt$ $\Rightarrow$ $\ln(5+v) - \ln(5-v) = 4t + c$ when $t = 0$, $v = 0$, $\Rightarrow 0 = 4 \times 0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln\left(\frac{5+v}{5-v}\right) = 4t$ $\Rightarrow$ $t = \frac{1}{4}\ln\left(\frac{5+v}{5-v}\right)$ *	 M1 E1 M1 A1 M1 A1 A1 E1 [8]	 for both A=1,B=1 separating variables correctly and indicating integration ft their A,B, condone absence of c ft finding c from an expression of correct form
(iv) When $t \to \infty$, $e^{-4t} \to 0$, $\Rightarrow$ $v \to 5/1 = 5$ when $t = 0.5$, $t = \frac{5(1 - e^{-2})}{1 + e^{-2}} = 3.8 \text{ m s}^{-1}$	E1 M1A1 [3]	
(v) The first model	E1 [1]	www

3 (i) $x = a(1 + kt)^{-1}$ $\Rightarrow$ $dx/dt = -ka(1 + kt)^{-2}$ $= -ka(x/a)^2$ $= -kx^2/a$ * OR $kt=a/x - 1$, $t= a/kx - 1/k$ $dt/dx= -a/kx^2$ $\Rightarrow dx/dt= -kx^2/a$	M1 A1 E1 [3] M1 A1 E1 [3]	Chain rule (or quotient rule) Substitution for x
(ii) When $t = 0$, $x = a \Rightarrow a = 2.5$ When $t = 1$, $x = 1.6 \Rightarrow 1.6 = 2.5/(1 + k)$ $\Rightarrow$ $1 + k = 1.5625$ $\Rightarrow$ $k = 0.5625$	B1 M1 A1 [3]	$a = 2.5$
(iii) In the long term, $x \to 0$	B1 [1]	or, for example, they die out.
(iv) $\frac{1}{2y-y^2} = \frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{2-y}$ $\Rightarrow$ $1 = A(2 - y) + By$ $y = 0 \Rightarrow 2A = 1 \Rightarrow A = ½$ $y = 2 \Rightarrow 1 = 2B \Rightarrow B = ½$ $\Rightarrow \frac{1}{2y-y^2} = \frac{1}{2y} + \frac{1}{2(2-y)}$	M1 M1 A1 A1 [4]	partial fractions evaluating constants by substituting values, equating coefficients or cover-up
(v) $\int \frac{1}{2y-y^2} dy = \int dt$ $\Rightarrow$ $\int [\frac{1}{2y} + \frac{1}{2(2-y)}] dy = \int dt$ $\Rightarrow$ ½ ln y – ½ ln$(2 - y) = t + c$ When $t = 0$, $y = 1 \Rightarrow 0 - 0 = 0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln y - \ln(2 - y) = 2t$ $\Rightarrow$ $\ln \frac{y}{2-y} = 2t$ * $\frac{y}{2-y} = e^{2t}$ $\Rightarrow$ $y = 2e^{2t} - ye^{2t}$ $\Rightarrow$ $y + ye^{2t} = 2e^{2t}$ $\Rightarrow$ $y(1 + e^{2t}) = 2e^{2t}$ $\Rightarrow$ $y = \frac{2e^{2t}}{1+e^{2t}} = \frac{2}{1+e^{-2t}}$ *	M1 B1 ft A1 E1 M1 DM1 E1 [7]	Separating variables ½ ln y – ½ ln$(2 - y)$ ft their A,B evaluating the constant Anti-logging Isolating y
(vi) As $t \to \infty$ $e^{-2t} \to 0 \Rightarrow y \to 2$ So long term population is 2000	B1 [1]	or $y = 2$