Edexcel Maths C4

Topic Questions from Papers

Rates & Differential Equations

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8. Liquid is pouring into a container at a constant rate of $20\text{ cm}^3\text{ s}^{-1}$ and is leaking out at a rate proportional to the volume of liquid already in the container.

(a) Explain why, at time t seconds, the volume, $V\text{ cm}^3$, of liquid in the container satisfies the differential equation

$$\frac{dV}{dt} = 20 - kV,$$

where k is a positive constant.

(2)

The container is initially empty.

(b) By solving the differential equation, show that

$$V = A + Be^{-kt},$$

giving the values of A and B in terms of k.

(6)

Given also that $\frac{dV}{dt} = 10$ when $t = 5$,

(c) find the volume of liquid in the container at 10 s after the start.

(5)

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Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

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7. The volume of a spherical balloon of radius r cm is $V\,\text{cm}^3$, where $V = \frac{4}{3}\pi r^3$.

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$.

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1000}{(2t+1)^2}, \qquad t \geqslant 0.$$

(b) Using the chain rule, or otherwise, find an expression in terms of r and t for $\dfrac{\mathrm{d}r}{\mathrm{d}t}$.

(2)

(c) Given that $V = 0$ when $t = 0$, solve the differential equation $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{1000}{(2t+1)^2}$, to obtain V in terms of t.

(4)

(d) Hence, at time $t = 5$,

(i) find the radius of the balloon, giving your answer to 3 significant figures,

(3)

(ii) show that the rate of increase of the radius of the balloon is approximately $2.90 \times 10^{-2}\,\text{cm s}^{-1}$.

(2)

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Question 7 continued

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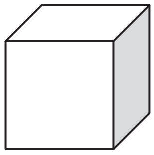
7.



At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^2 s^{-1}.

Show that

(a) $\dfrac{dx}{dt} = \dfrac{k}{x}$, where k is a constant to be found,

(4)

(b) $\dfrac{dV}{dt} = 2V^{\frac{1}{3}}$.

(4)

Given that $V = 8$ when $t = 0$,

(c) solve the differential equation in part (b), and find the value of t when $V = 16\sqrt{2}$.

(7)

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Question 7 continued

Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

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4. (a) Express $\dfrac{2x-1}{(x-1)(2x-3)}$ in partial fractions.

(3)

(b) Given that $x \geqslant 2$, find the general solution of the differential equation

$$(2x-3)(x-1)\frac{dy}{dx} = (2x-1)y\,.$$

(5)

(c) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = f(x)$.

(4)

Leave blank

8. A population growth is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP ,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(a) solve the differential equation, giving P in terms of P_0, k and t.

(4)

Given also that $k = 2.5$,

(b) find the time taken, to the nearest minute, for the population to reach $2P_0$.

(3)

In an improved model the differential equation is given as

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \lambda P \cos \lambda t ,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(c) solve the second differential equation, giving P in terms of P_0, λ and t.

(4)

Given also that $\lambda = 2.5$,

(d) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model.

(3)

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Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

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8. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $cm^3\,s^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.}$$

(3)

When $h = 25$, water is leaking out of the hole at 400 $cm^3\,s^{-1}$.

(b) Show that $k = 0.02$

(1)

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\,dh.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}}\,dh$.

(6)

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

(1)

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Question 8 continued

Q8

(Total 13 marks)

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3.

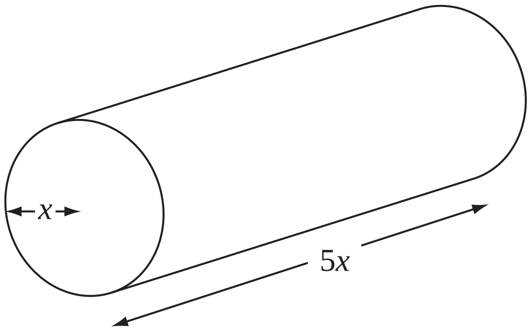

Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm.
The cross-sectional area of the rod is increasing at the constant rate of 0.032 cm^2 s^{-1}.

(a) Find $\frac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate of increase of the volume of the rod when $x = 2$.

(4)

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Question 3 continued

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7. (a) Express $\dfrac{2}{4-y^2}$ in partial fractions.

(3)

(b) Hence obtain the solution of

$$2\cot x \frac{dy}{dx} = (4-y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$.

(8)

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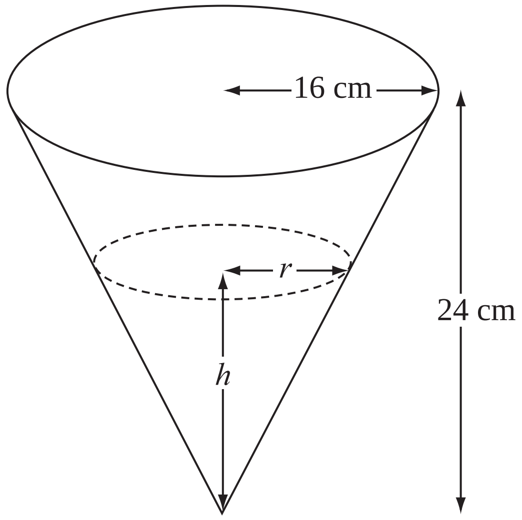

Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \dfrac{4\pi h^3}{27}$.

(2)

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of 8 cm^3 s^{-1}.

(b) Find, in terms of π, the rate of change of h when $h = 12$.

(5)

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Question 5 continued

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5. (a) Find $\int \frac{9x+6}{x}\,\mathrm{d}x,\ x>0.$

(2)

(b) Given that $y=8$ at $x=1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(9x+6)\,y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2=\mathrm{g}(x)$.

(6)

N 3 5 3 8 2 A 0 1 6 2 8

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Question 5 continued

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6. The area A of a circle is increasing at a constant rate of $1.5\,\text{cm}^2\,\text{s}^{-1}$. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is $2\,\text{cm}^2$.

(5)

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6.

$$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta)\,d\theta$.

(7)

H 3 5 3 8 6 A 0 2 0 3 2

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8.

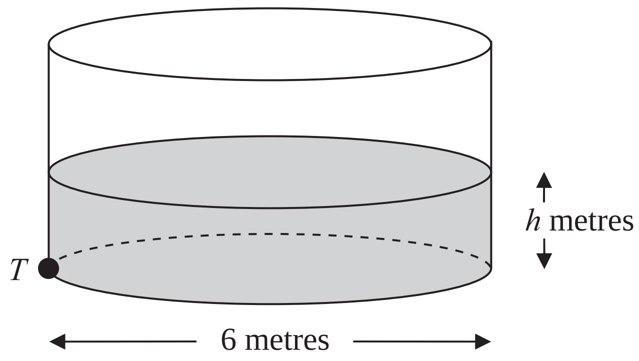

Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(a) Show that t minutes after the tap has been opened

$$75\frac{\mathrm{d}h}{\mathrm{d}t} = (4 - 5h)$$

(5)

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$

(6)

Leave blank

Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

H 3 5 3 8 6 A 0 3 1 3 2

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2. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geqslant 0$$

Use differentiation to find the value of $\frac{dI}{dt}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(5)

Leave blank

3. (a) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(b) Hence find $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x$, where $x>1$.

(3)

(c) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{\mathrm{d}y}{\mathrm{d}x}=5y, \quad x>1,$$

for which $y=8$ at $x=2$. Give your answer in the form $y=\mathrm{f}(x)$.

(6)

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Question 3 continued

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3.

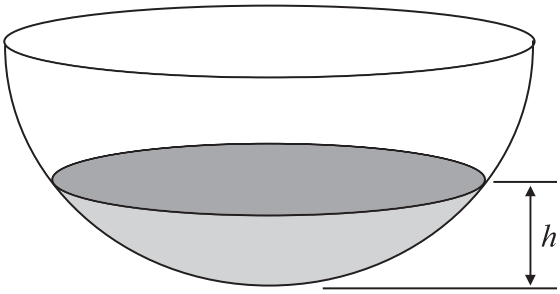

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl. When the depth of the water is h m, the volume $V\,\text{m}^3$ is given by

$$V = \frac{1}{12}\pi h^2(3-4h), \qquad 0 \leqslant h \leqslant 0.25$$

(a) Find, in terms of π, $\frac{dV}{dh}$ when $h = 0.1$

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800}\ \text{m}^3\text{s}^{-1}$.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$

(2)

Leave blank

8. (a) Find $\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}y$

(2)

(b) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{(4y+3)}}{x^2}$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

Leave blank

8. (a) Express $\dfrac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{15}P(5-P), \qquad t \geqslant 0$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where a, b and c are integers.

(8)

(c) Hence show that the population cannot exceed 5000

(1)

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Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

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2.

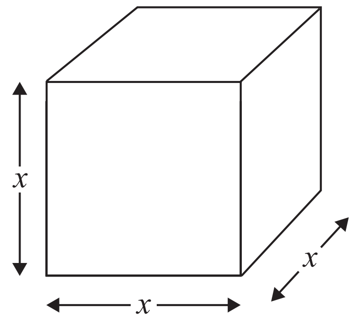

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.
At time t seconds, the length of each edge of the cube is x cm,
and the volume of the cube is V cm^3.

(a) Show that $\dfrac{dV}{dx} = 3x^2$ **(1)**

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^{3}s^{-1},

(b) find $\dfrac{dx}{dt}$, when $x = 8$ **(2)**

(c) find the rate of increase of the total surface area of the cube, in cm^{2}s^{-1}, when $x = 8$ **(3)**

Leave blank

Question 2 continued

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4. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}$$

(5)

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8. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3 °C and t minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is θ °C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation,

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{(3-\theta)}{125}$$

(a) By solving the differential equation, show that,

$$\theta = A\mathrm{e}^{-0.008t} + 3$$

where A is a constant.

(4)

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16 °C,

(b) find the time taken for the temperature of the water in the bottle to fall to 10 °C, giving your answer to the nearest minute.

(5)

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Question 8 continued

Q8

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

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8. In an experiment testing solid rocket fuel, some fuel is burned and the waste products are collected. Throughout the experiment the sum of the masses of the unburned fuel and waste products remains constant.

Let x be the mass of waste products, in kg, at time t minutes after the start of the experiment. It is known that at time t minutes, the rate of increase of the mass of waste products, in kg per minute, is k times the mass of unburned fuel remaining, where k is a positive constant.

The differential equation connecting x and t may be written in the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(M - x), \text{ where } M \text{ is a constant.}$$

(a) Explain, in the context of the problem, what $\frac{\mathrm{d}x}{\mathrm{d}t}$ and M represent.

(2)

Given that initially the mass of waste products is zero,

(b) solve the differential equation, expressing x in terms of k, M and t.

(6)

Given also that $x = \frac{1}{2}M$ when $t = \ln 4$,

(c) find the value of x when $t = \ln 9$, expressing x in terms of M, in its simplest form.

(4)

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Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

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6. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

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Question 6 continued

P 4 3 1 3 7 A 0 2 1 3 2

Turn over

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration **(+ *constant*)**

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x+\cot x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x+\tfrac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$