Edexcel Maths C4

Topic Questions from Papers

Coordinate Geometry

&

Parametric Differentiation

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6. A curve has parametric equations

$$x = 2\cot t, \quad y = 2\sin^2 t, \quad 0 < t \leqslant \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t.

(4)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

(4)

(c) Find a cartesian equation of the curve in the form $y = f(x)$. State the domain on which the curve is defined.

(4)

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Question 6 continued

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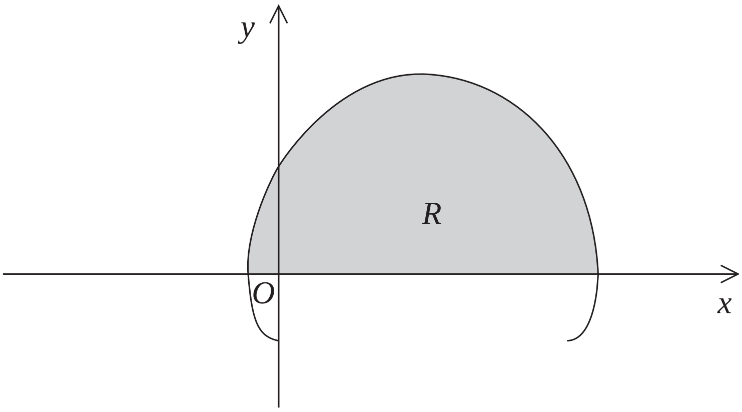
8. **Figure 2**



The curve shown in Figure 2 has parametric equations

$$x = t - 2\sin t, \qquad y = 1 - 2\cos t, \qquad 0 \leqslant t \leqslant 2\pi.$$

(a) Show that the curve crosses the x-axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x-axis, as shown shaded in Figure 2.

(b) Show that the area of R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, \mathrm{d}t.$$

(3)

(c) Use this integral to find the exact value of the shaded area.

(7)

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Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

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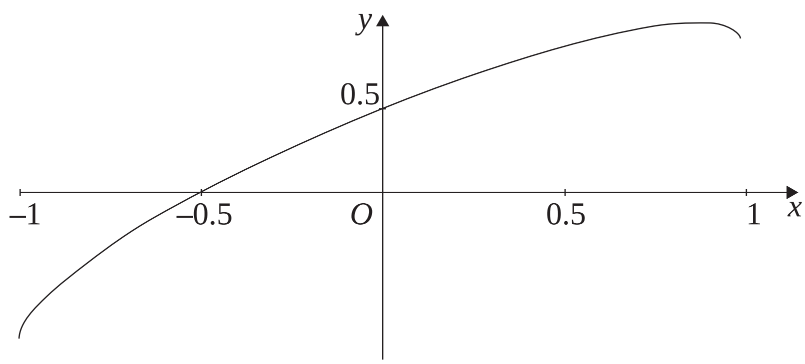
4. **Figure 2**



The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right), \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

(6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \qquad -1 < x < 1.$$

(3)

N 2 3 5 6 3 A 0 8 2 0

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Question 4 continued

Q4

(Total 9 marks)

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6. A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.

(3)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined.

(5)

(c) Find a cartesian equation of the curve in the form $y^2 = f(x)$.

(4)

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Question 6 continued

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7.

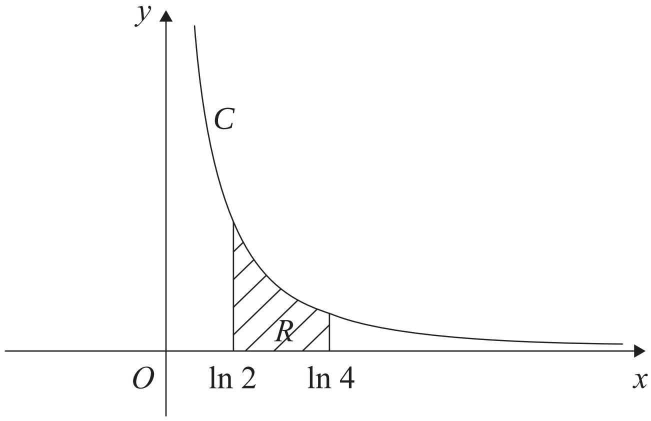

Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

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Question 7 continued

8.

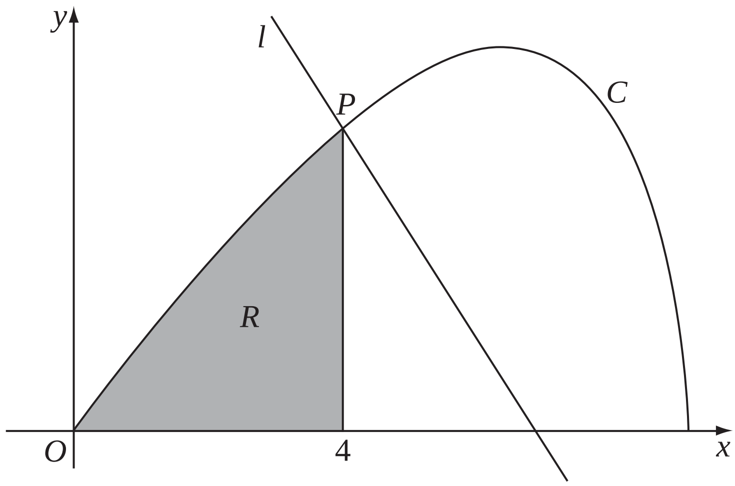

Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t \, dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

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Question 8 continued

Q8

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

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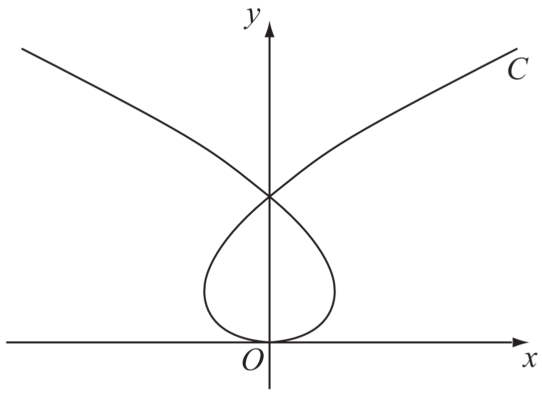

Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)

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Question 7 continued

Q7

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

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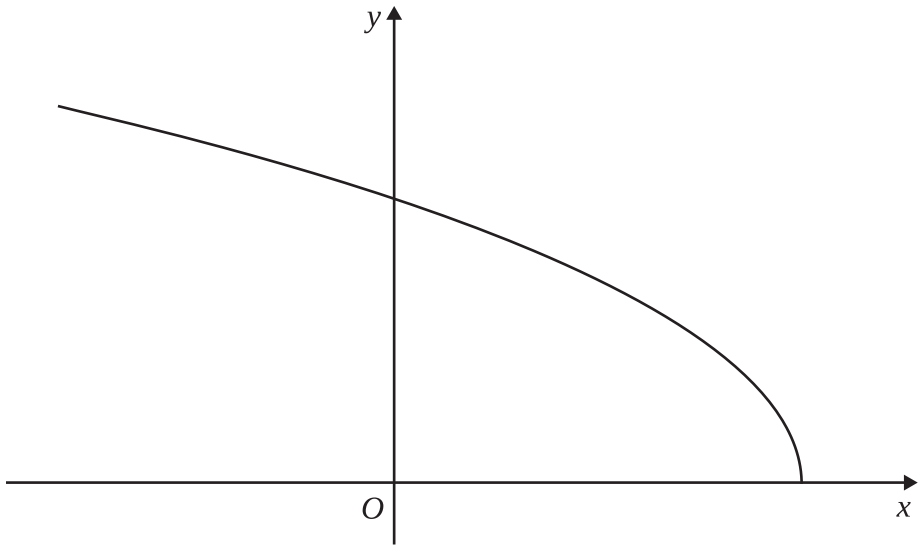

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \qquad y = 6\sin t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \dfrac{\pi}{3}$.

(4)

(b) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant k.

(4)

(c) Write down the range of $\mathrm{f}(x)$.

(2)

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Question 5 continued

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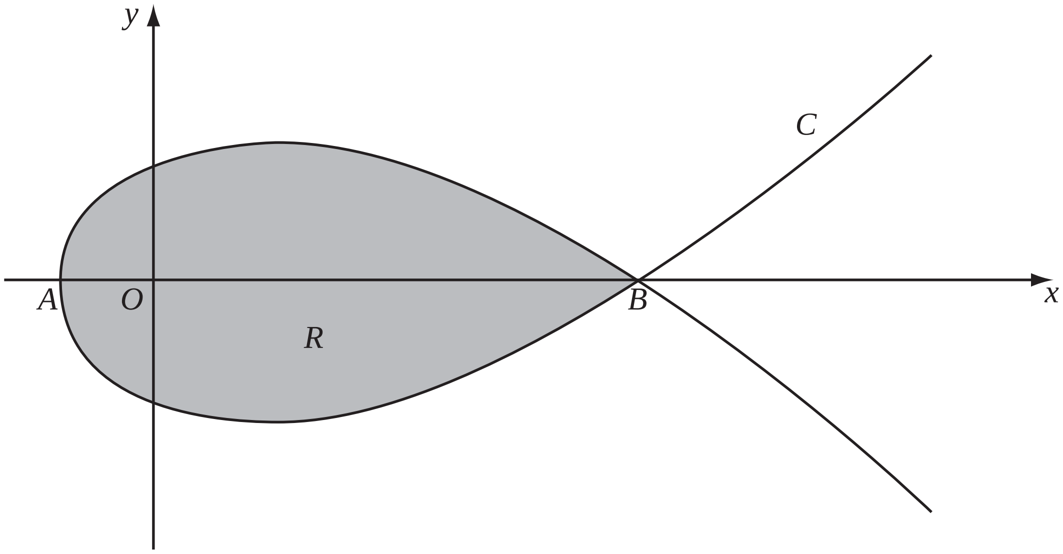

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

(3)

The region R, as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)

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Question 7 continued

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4. A curve C has parametric equations

$$x = \sin^2 t, \quad y = 2\tan t, \quad 0 \leqslant t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t.

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P.

(6)

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Question 4 continued

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6. The curve C has parametric equations

$$x = \ln t, \quad y = t^2 - 2, \quad t > 0$$

Find

(a) an equation of the normal to C at the point where $t = 3$,

(6)

(b) a cartesian equation of C.

(3)

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Question 6 continued

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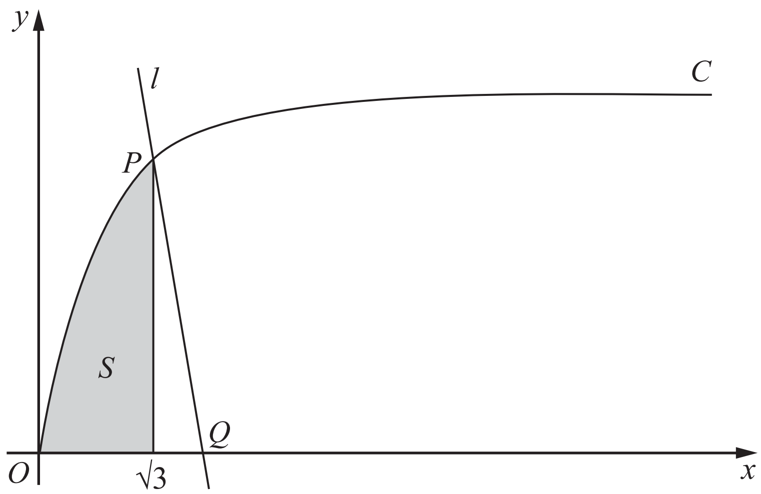

Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

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Question 7 continued

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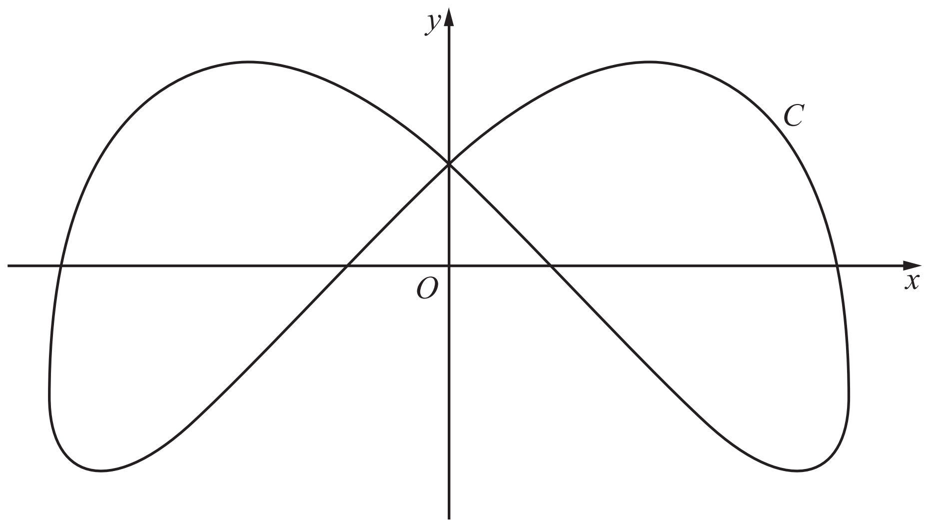

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(3)

(b) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$

(5)

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Question 5 continued

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Question 6 continued

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5.

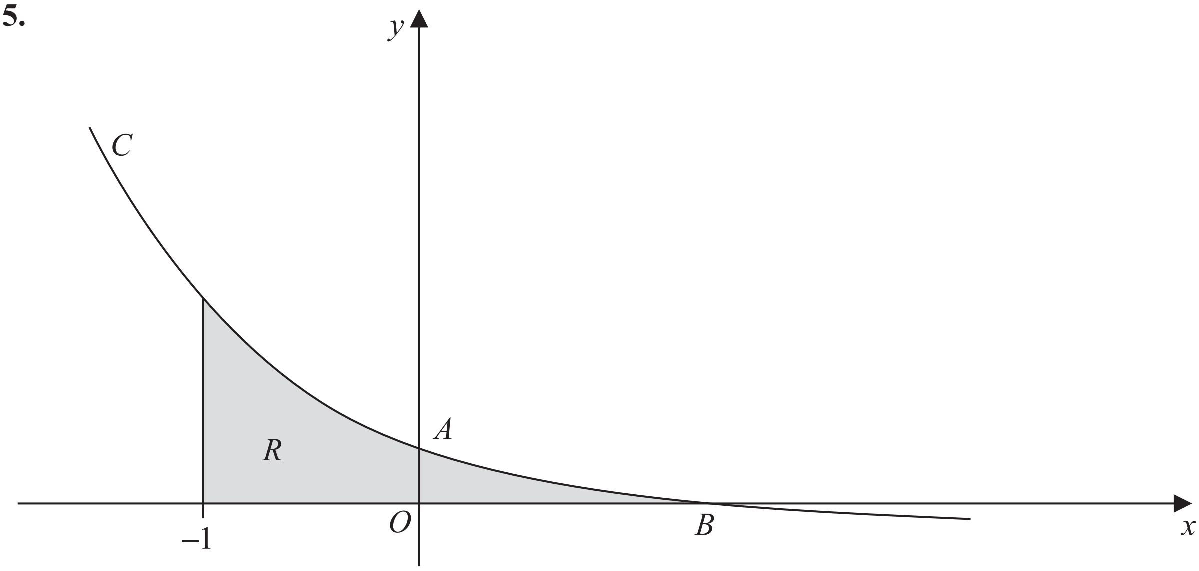

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).

(2)

(b) Find the x coordinate of the point B.

(2)

(c) Find an equation of the normal to C at the point A.

(5)

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R.

(6)

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Question 5 continued

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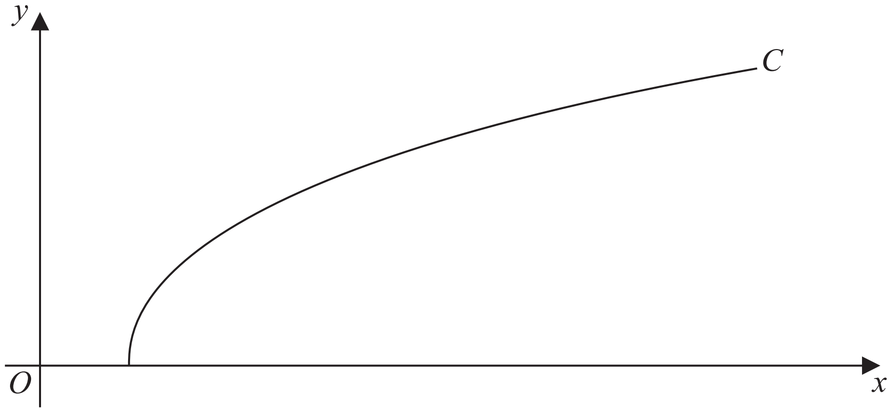

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 27\sec^3 t, \quad y = 3\tan t, \qquad 0 \leqslant t \leqslant \frac{\pi}{3}$$

(a) Find the gradient of the curve C at the point where $t = \frac{\pi}{6}$

(4)

(b) Show that the cartesian equation of C may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \qquad a \leqslant x \leqslant b$$

stating the values of a and b.

(3)

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Question 7 continued

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4. A curve C has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ at the point where $t = \frac{\pi}{6}$

(4)

(b) Find a cartesian equation for C in the form

$$y = f(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant k.

(3)

(c) Write down the range of f(x).

(2)

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Question 4 continued

P 4 3 1 3 7 A 0 1 3 3 2

Turn over

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration **(+ *constant*)**

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x+\cot x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x+\tfrac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$