1. (a) Find $\int \frac{9x+6}{x}\,dx, x > 0.$

(2)

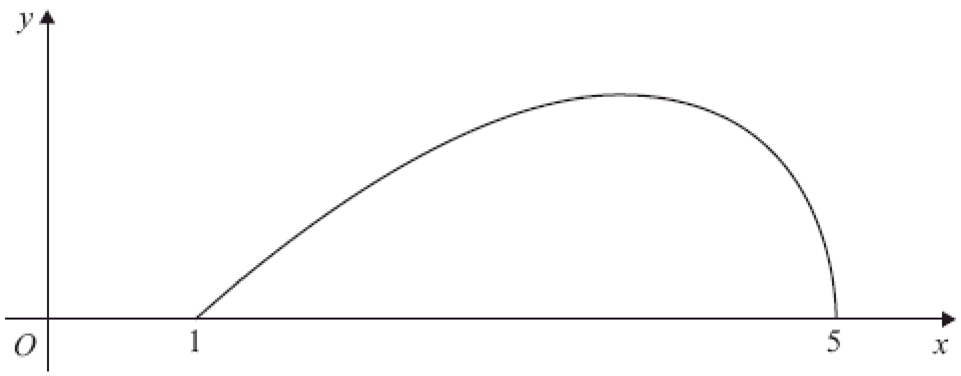
(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$.

(6)
(Total 8 marks)

2. (a) Find $\int \sqrt{(5-x)}\,dx$



(2)

The diagram above shows a sketch of the curve with equation

$$y = (x - 1)\sqrt{(5 - x)}, \qquad 1 \le x \le 5$$

(b) (i) Using integration by parts, or otherwise, find

$$\int (x-1)\sqrt{(5-x)}\,dx$$

(4)

(ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\,dx$.

(2)
(Total 8 marks)

3. (a) Find $\int \tan^2 x\,dx$.

(2)

(b) Use integration by parts to find $\int \frac{1}{x^3} \ln x\,dx$.

(4)

(c) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x}\,dx = \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k,$$

where k is a constant.

(7)
(Total 13 marks)

1.	(a)	$\int \frac{9x+6}{x}dx = \int\left(9+\frac{6}{x}\right)dx$		M1	
		$=9x + 6\ln x\ (+C)$		A1	2
	(b)	$\int \frac{1}{y^{\frac{1}{3}}}dy = \int \frac{9x+6}{x}dx$	Integral signs not necessary	B1	
		$\int y^{-\frac{1}{3}}dy = \int \frac{9x+6}{x}dx$			
		$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x(+C)$	$\pm ky^{\frac{2}{3}} =$ their (a)	M1	
		$\frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x(+C)$	ft their (a)	A1ft	

$y = 8, x = 1$

$\frac{3}{2}8^{\frac{2}{3}} = 9 + 6\ln 1 + C$ M1

$C = -3$ A1

$y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$

$y^2 = (6x + 4\ln x - 2)^3 \quad \left(= 8(3x + 2\ln x - 1)^3\right)$ A1 6

[8]

2. (a) $\int \sqrt{(5-x)}\,dx = \int (5-x)^{\frac{1}{2}}\,dx = \frac{(5-x)^{\frac{1}{2}}}{-\frac{3}{2}}(+C)$ M1 A1 2

$\left(= -\frac{2}{3}(5-x)^{\frac{3}{2}} + C\right)$

(b) (i) $\int (x-1)\sqrt{(5-x)}\,dx = -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3}\int (5-x)^{\frac{3}{2}}\,dx$ M1 A1ft

$= \quad \ldots \quad + \frac{2}{3} \times \frac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}}(+C)$ M1

$-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}}\ (+C)$ A1 4

(ii) $\left[-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}}\right]_1^5 = (0-0) - \left(0 - \frac{4}{15} \times 4^{\frac{5}{2}}\right)$

$= \frac{128}{15}\left(= 8\frac{8}{15} \approx 8.53\right)$ awrt 8.53 M1 A1 2

Alternatives for (b)

$u^2 = 5 - x \Rightarrow 2u\frac{du}{dx} = -1 \left(\Rightarrow \frac{dx}{du} = -2u\right)$

$\int (x-1)\sqrt{(5-x)}\,dx = \int (4-u^2)u\frac{dx}{du}\,du = \int (4-u^2)\,u\,(-2u)\,du$ M1 A1

$= \int (2u^4 - 8u^2)\,du = \frac{2}{5}u^5 - \frac{8}{3}u^3 (+C)$ M1

$$=\frac{2}{5}(5-x)^{\frac{5}{2}}-\frac{8}{3}(5-x)^{\frac{3}{2}}\ (+C)$$ A1

Alternatives for (c)

$x = 1 \Rightarrow u = 2,\ x = 5 \Rightarrow u = 0$

$$\left[\frac{2}{5}u^5-\frac{8}{3}u^3\right]_2^0=(0-0)-\left(\frac{64}{5}-\frac{64}{3}\right)$$ M1

$$=\frac{128}{15}\left(=8\frac{8}{15}\approx 8.53\right)$$ awrt 8.53 A1 2

[8]

3. (a) $\int \tan^2 x\,dx$

[*NB* : <u>$\sec^2 A = 1 + \tan^2 A$</u> gives <u>$\tan^2 A = \sec^2 A - 1$</u>] The correct <u>underlined identity</u>. M1 oe

$$=\int \sec^2 x - 1\,dx$$

= <u>$\tan x - x (+c)$</u> Correct integration with/without + c A1 2

(b) $\int \frac{1}{x^3}\ln x\,dx$

$$\left\{\begin{array}{l} u=\ln x \quad \Rightarrow \quad \frac{du}{dx}=\frac{1}{x} \\ \frac{dv}{dx}=x^{-3} \Rightarrow v=\frac{x^{-2}}{-2}=\frac{-1}{2x^2}\end{array}\right\}$$

$$=-\frac{1}{2x^2}\ln x-\int -\frac{1}{2x^2}.\frac{1}{x}\,dx$$ Use of 'integration by parts' formula in the correct direction. M1

Correct direction means that $u = \ln x$.

Correct expression. A1

$$=-\frac{1}{2x^2}\ln x+\frac{1}{2}\int\frac{1}{x^3}\,dx$$ An attempt to multiply through $\frac{k}{x^n}$, $n \in$ □ $n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...

$$=-\frac{1}{2x^2}\ln x+\frac{1}{2}\left(-\frac{1}{2x^2}\right)(+c)$$ ... "integrate"(process the result); M1

<u>correct solution</u> with/without + c A1 oe 4

(c) $\int \frac{e^{3x}}{1+e^x}\,dx$

$\left\{u = 1 + e^x \Rightarrow \underline{\frac{du}{dx} = e^x}, \underline{\frac{dx}{du} = \frac{1}{e^x}}, \underline{\frac{dx}{du} = \frac{1}{u-1}}\right\}$ Differentiating to find any one of the three underlined — B1

$\int \frac{e^{2x}.e^x}{1+e^x}\,dx = \int \frac{(u-1)^2.e^x}{u}.\frac{1}{e^x}\,du$ Attempt to substitute for $e^{2x} = f(u)$, their $\frac{dx}{du} = \frac{1}{e^x}$ and $u = 1 + e^x$

or = $\int \frac{(u-1)^3}{u}.\frac{1}{(u-1)}\,du$ or $e^{3x} = f(u)$, their $\frac{dx}{du} = \frac{1}{u-1}$ and $u = 1 + e^x$. — M1 *

$= \int \frac{(u-1)^2}{u}\,du$ $\underline{\int \frac{(u-1)^2}{u}\,du}$ — A1

$= \int \frac{u^2 - 2u + 1}{u}\,du$ An attempt to multiply out their numerator to give at least three terms

$= \int u - 2 + \frac{1}{u}\,du$ and divide through each term by u — dM1 *

$= \frac{u^2}{2} - 2u + \ln u\ (+c)$ Correct integration with/without +c — A1

$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$ Substitutes $u = 1 + e^x$ back into their integrated expression with at least two terms. — dM1 *

$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1 + e^x) + c$

$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1 + e^x) + c$

$= \frac{1}{2}e^{2x} - e^x + \ln(1 + e^x) - \frac{3}{2} + c$

$= \frac{1}{2}e^{2x} - e^x + \ln(1 + e^x) + k$ **AG** $\underline{\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k}$ must use a + c + and "$-\frac{3}{2}$" combined. — A1 cso — 7

[13]

1. Part (a) of this question proved awkward for many. The integral can be carried out simply by decomposition, using techniques available in module C1. It was not unusual to see integration by parts attempted. This method will work if it is known how to integrate ln x , but this requires a further integration by parts and complicates the question unnecessarily. In part (b), most could separate the variables correctly but the integration of $\frac{1}{y^{\frac{1}{3}}}$, again a C1 topic, was frequently incorrect.

Weakness in algebra sometimes caused those who could otherwise complete the question to lose the last mark as they could not proceed from $y^{\frac{2}{3}} = 6x + 41n\,x - 2$ to $y^2 = (6x + 41\text{n}\,x - 2)^3$. Incorrect answers, such as $y^2 = 216x^3 + 64 \ln x^3 - 8$, were common in otherwise correct solutions.

2. Throughout this question sign errors were particularly common. In part (a), nearly all recognised that $(5-x)^{\frac{3}{2}}$ formed part of the answer, and this gained the method mark, but $\frac{3}{2}(5-x)^{\frac{3}{2}}, -\frac{3}{2}(5-x)^{\frac{3}{2}}$ and $\frac{2}{3}(5-x)^{\frac{3}{2}}$, instead of the correct $-\frac{2}{3}(5-x)^{\frac{3}{2}}$, were all frequently seen. Candidates who made these errors could still gain 3 out of the 4 marks in part (b)(i) if they proceeded correctly. Most candidates integrated by parts the "right way round" and were able to complete the question. Further sign errors were, however, common.

3. In part (a), a surprisingly large number of candidates did not know how to integrate $\tan^2 x$. Examiners were confronted with some strange attempts involving either double angle formulae or logarithmic answers such as $\ln(\sec^2 x)$ or $\ln(\sec^4 x)$. Those candidates who realised that the needed the identity $\sec^2 x = 1 + \tan^2 x$ sometimes wrote it down incorrectly.

Part (b) was probably the best attempted of the three parts in the question. This was a tricky integration by parts question owing to the term of $\frac{1}{x^3}$, meaning that candidates had to be especially careful when using negative powers. Many candidates applied the integration by parts formula correctly and then went on to integrate an expression of the form $\frac{k}{x^3}$ to gain 3 out of the 4 marks available. A significant number of candidates failed to gain the final accuracy mark owing to sign errors or errors with the constants α and β in $\frac{\alpha}{x^2}\ln x + \frac{\beta}{x^2} + c.$ A minority of candidates applied the by parts formula in the 'wrong direction' and incorrectly stated that $\frac{dv}{dh}$ = ln x implied $v = \frac{1}{x}$.

In part (c), most candidates correctly differentiated the substitution to gain the first mark. A significant proportion of candidates found the substitution to obtain an integral in terms of u more demanding. Some candidates did not realise that e^{2x} and e^{3x} are $(e^x)^2$ and $(e^x)^3$ respectively and hence $u^2 - 1$, rather than $(u-1)^2$ was a frequently encountered error seen in the numerator of the substituted expression. Fewer than half of the candidates simplified their substituted expression to arrive at the correct result of $\int \frac{(u-1)^2}{u}\,du.$ Some candidates could not proceed further at this point but the majority of the candidates who achieved this result were able to multiply out the numerator, divide by u, integrate and substitute back for u. At this point some candidates struggled to achieve the expression required. The most common misconception was that the constant of integration was a fixed constant to be determined, and so many candidates concluded that $k = -\frac{3}{2}$. Many candidates did not realise that $-\frac{3}{2}$ when added to c combined to make another arbitrary constant k.