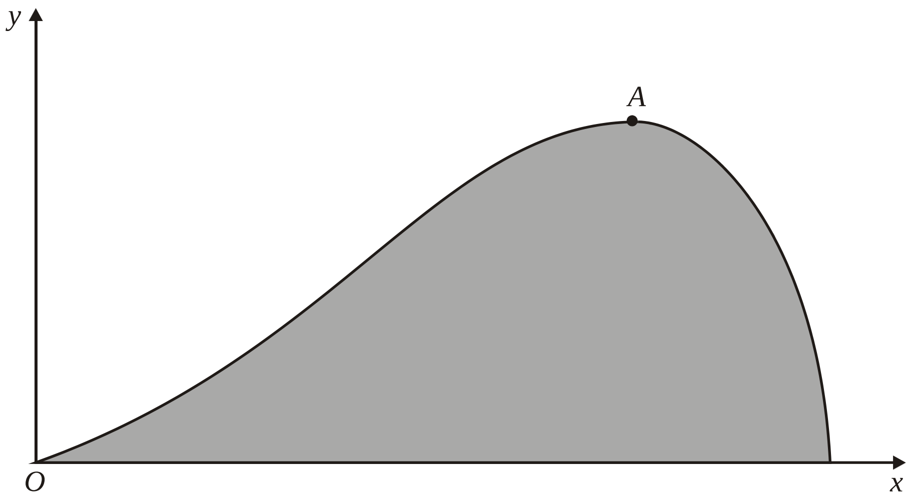
1. A curve has equation

$$x^2 + 2xy - 3y^2 + 16 = 0.$$

Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$.

(Total 7 marks)

2.



The diagram above shows a graph of $y = x\sqrt{\sin x}$, $0 < x < \pi$. The maximum point on the curve is A.

(a) Show that the x-coordinate of the point A satisfies the equation $2\tan x + x = 0$.

(4)

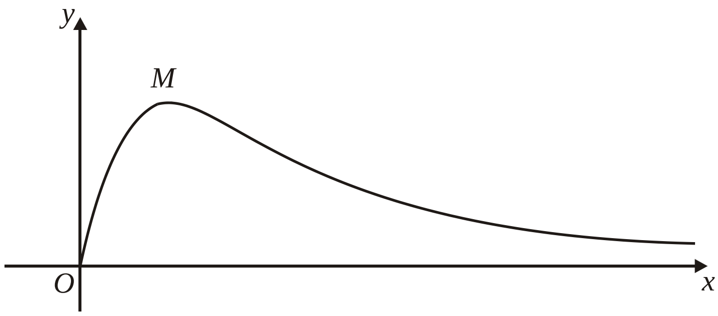
The finite region enclosed by the curve and the x-axis is shaded as shown in the diagram above.

A solid body S is generated by rotating this region through 2π radians about the x-axis.

(b) Find the exact value of the volume of S.

(7)
(Total 11 marks)

3.



The diagram above shows the curve with equation $y = x^{\frac{1}{2}} e^{-2x}$.

(a) Find the x-coordinate of M, the maximum point of the curve.

(5)

The finite region enclosed by the curve, the x-axis and the line $x = 1$ is rotated through 2π about the x-axis.

(b) Find, in terms of π and e, the volume of the solid generated.

(7)
(Total 12 marks)

1. $2x + \left(2x\frac{dy}{dx} + 2y\right) - 6y\frac{dy}{dx} = 0$ M1 (A1) A1

$\frac{dy}{dx} = 0 \Rightarrow x + y = 0$ (or equivalent) M1

Eliminating either variable and solving for at least one value of x or y. M1

$y^2 - 2y^2 - 3y^2 + 16 = 0$ or the same equation in x

$y = \pm 2$ or $x = \pm 2$ A1

$(2 - 2), (-2, 2)$ A1 7

Note: $\frac{dy}{dx} = \frac{x + y}{3y - x}$

Alternative:

$3y^2 - 2xy - (x^2 + 16) = 0$

$y = \frac{2x \pm \sqrt{(16x^2 + 192)}}{6}$

$\frac{dy}{dx} = \frac{1}{3} \pm \frac{1}{3}.\frac{8x}{\sqrt{(16x^2 + 192)}}$ M1 A1 ± A1

$\frac{dy}{dx} = 0 \Rightarrow \frac{8x}{\sqrt{(16x^2 + 192)}} = \pm 1$ M1

		$64x^2 = 16x^2 + 192$	M1 A1	
		$x = \pm 2$	A1	7
		$(2, -2), (-2, 2)$		

[7]

2.	(a)	$\frac{dy}{dx} = \sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}}\cos x$	M1, A1	
		At A $\sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}}\cos x = 0$	dM1	
		$\therefore \sin x + \frac{x}{2}\cos x = 0$ (essential to see intermediate line before given answer)		
		$\therefore 2\tan x + x = 0$ (*)	A1	4
	(b)	$V = \pi\int y^2 dx = \pi\int x^2 \sin x dx$	M1	
		$= \pi\left[-x^2\cos x + \int 2x\cos x dx\right]_0^\pi$	M1 A1	
		$= \pi\left[-x^2\cos x + 2x\sin x - \int 2\sin x dx\right]_0^\pi$	M1	
		$= \pi\left[-x^2\cos x + 2x\sin x + 2\cos x\right]_0^\pi$	A1	
		$= \pi[\pi^2 - 2 - 2]$	M1	
		$= \pi[\pi^2 - 4]$	A1	7

[11]

3.	(a)	$\frac{dy}{dx} = 2e^{-2x}\sqrt{x} + \frac{e^{-2x}}{2\sqrt{x}}$	M1 A1 A1	
		Putting $\frac{dy}{dx} = 0$ and attempting to solve	dM1	
		$x = \frac{1}{4}$	A1	5
	(b)	Volume = $\pi\int_0^1 (\sqrt{x}e^{-2x})^2 dx = \pi\int_0^1 xe^{-4x} dx$	M1 A1	
		$\int xe^{-4x} dx = -\frac{1}{4}xe^{-4x} + \int \frac{1}{4}e^{-4x} dx$	M1 A1	
		$= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}$	A1 ft	
		Volume = $\pi\left[-\frac{1}{4}e^{-4} - \frac{1}{16}e^{-4}\right] - \left[-\frac{1}{16}\right] = \frac{\pi}{16}[1 - 5e^{-4}]$	M1 A1	7

[12]

1. Almost all candidates could start this question and the majority could differentiate implicitly correctly. This is an area of work which has definitely improved in recent years. Many, having found $\frac{dy}{dx}$, could not use it and it was disturbing to find a substantial number of students in this relatively advanced A2 module proceeding from $\frac{x+y}{3y-x}=0$ to $x + y = 3y - x$. Those who did obtain $y = -x$ often went no further but those who did could usually obtain both correct points, although extra incorrect points were often seen.

2. Most candidates made some attempt to differentiate $x\sqrt{\sin x}$, with varying degrees of success. $\sqrt{\sin x} + x\sqrt{\cos x}$ was the most common wrong answer. Having struggled with the differentiation, several went no further with this part. It was surprising to see many candidates with a correct equation who were not able to tidy up the √ terms to reach the required result.

Most candidates went on to make an attempt at $\int \pi y^2 \, dx$. The integration by parts was generally well done, but there were many of the predictable sign errors, and several candidates were clearly not expecting to have to apply the method twice in order to reach the answer. A lot of quite good candidates did not get to the correct final answer, as there were a number of errors when substituting the limits.

3. This question involved differentiation using the product rule in part (a) and integration using parts in part (b). It was answered well, with most of the difficulties being caused by the use of indices and the associated algebra. Some candidates wasted time in part (a) by finding the y-coordinate which was not requested. A sizeable proportion of the candidates misquoted the formula for volume of revolution.