1. A curve C has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (3, 2).

(Total 7 marks)

2. The curve C has the equation

$$\cos 2x + \cos 3y = 1, \qquad -\frac{\pi}{4} \le x \le \frac{\pi}{4},\ 0 \le y \le \frac{\pi}{6}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

The point P lies on C where $x = \frac{\pi}{6}$.

(b) Find the value of y at P.

(3)

(c) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers.

(3)
(Total 9 marks)

3. The curve C has the equation $ye^{-2x} = 2x + y^2$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P on C has coordinates (0, 1).

(b) Find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)
(Total 9 marks)

4. A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)
(Total 7 marks)

5. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(6)

(b) Find the coordinates of P and Q.

(3)
(Total 9 marks)

6. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)
(Total 9 marks)

7. A set of curves is given by the equation $\sin x + \cos y = 0.5$.

(a) Use implicit differentiation to find an expression for $\frac{dy}{dx}$.

(2)

For $-\pi < x < \pi$ and $-\pi < y < \pi$,

(b) find the coordinates of the points where $\frac{dy}{dx} = 0$.

(5)
(Total 7 marks)

8. A curve C is described by the equation

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0.$$

Find an equation of the normal to C at the point (0, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(Total 7 marks)

9. A curve C is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0.$$

Find an equation of the tangent to C at the point (1, –2), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(Total 7 marks)

10. The value £V of a car t years after the 1st January 2001 is given by the formula

$$V = 10\,000 \times (1.5)^{-t}.$$

(a) Find the value of the car on 1st January 2005.

(2)

(b) Find the value of $\frac{dV}{dt}$ when $t = 4$.

(3)

(c) Explain what the answer to part (b) represents.

(1)
(Total 6 marks)

11. A curve has equation

$$x^2 + 2xy - 3y^2 + 16 = 0.$$

Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$.

(Total 7 marks)

12. The curve C has equation $5x^2 + 2xy - 3y^2 + 3 = 0$. The point P on the curve C has coordinates (1, 2).

(a) Find the gradient of the curve at P.

(5)

(b) Find the equation of the normal to the curve C at P, in the form $y = ax + b$, where a and b are constants.

(3)
(Total 8 marks)

= 0.

(Total 7 marks)

13. A curve has equation

$$x^3 - 2xy - 4x + y^3 - 51 = 0.$$

Find an equation of the normal to the curve at the point (4, 3), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(Total 8 marks)

1. $\frac{d}{dx}\left(2^x\right) = \ln 2.2^x$ B1

$\ln 2.2^x + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$ M1 A1 = A1

Substituting $(3, 2)$

$8\ln 2 + 4\frac{dy}{dx} = 4 + 6\frac{dy}{dx}$ M1

$\frac{dy}{dx} = 4\ln 2 - 2$ Accept exact equivalents M1 A1 7

[7]

2. (a) $-2\sin 2x - 3\sin 3y\frac{dy}{dx} = 0$ M1 A1

$\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 3y}$ Accept $\frac{2\sin 2x}{-3\sin 3y}$, $\frac{-2\sin 2x}{3\sin 3y}$ A1 3

(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6}\right) + \cos 3y = 1$ M1

$\cos 3y = \frac{1}{2}$ A1

$3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$ awrt 0.349 A1 3

(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{dy}{dx} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\frac{2}{3}$ M1

$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$ M1

Leading to $6x + 9y - 2\pi = 0$ A1 3

[9]

3. (a) $e^{-2x}\frac{dy}{dx} - 2ye^{-2x} = 2 + 2y\frac{dy}{dx}$ A1 correct RHS *M1 A1

$\frac{d}{dx}(ye^{-2x}) = e^{-2x}\frac{d}{dx} - 2ye^{-2x}$ B1

$(e^{-2x} - 2y)\frac{dy}{dx} = 2 + 2ye^{-2x}$ *M1

$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$ A1 5

(b) At P, $\frac{dy}{dx} = \frac{2 + 2e^{o}}{e^{o} - 2} = -4$ M1

Using $mm' = -1$

$m' = \frac{1}{4}$ M1

$y - 1 = \frac{1}{4}(x - 0)$ M1

$x - 4y + 4 = 0$ or any integer multiple A1 4

Alternative for (a) differentiating implicitly with respect to y.

$e^{-2x} - 2ye^{-2x}\frac{dx}{dy} = 2\frac{dx}{dy} + 2y$ A1 correct RHS *M1 A1

$\frac{d}{dy}(ye^{-2x}) = e^{-2x} - 2ye^{-2x}\frac{dx}{dy}$ B1

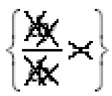
$(2 + 2ye^{-2x})\frac{dx}{dy} = e^{-2x} - 2y$ *M1

$\frac{dx}{dy} = \frac{e^{-2x} - 2y}{2 + 2ye^{-2x}}$

$\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$ A1 5

[9]

4. (a) $C: y^2 - 3y = x^3 + 8$ Differentiates implicitly to include either

 $2y\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$ $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.) M1

Correct equation. A1

A correct (condoning sign error) attempt to

$(2y - 3)\frac{dy}{dx} = 3x^2$ combine or factorise their '$2y\frac{dy}{dx} - 3\frac{dy}{dx}$'. M1

Can be implied.

$\frac{dy}{dx} = \frac{3x^2}{2y-3}$	$\underline{\frac{3x^2}{2y-3}}$	A1 oe	4
(b) $y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1	
$x^3 = -8 \Rightarrow \underline{x = -2}$		A1	
$(-2, 3) \Rightarrow \frac{dy}{dx} = \frac{3(4)}{6-3} \Rightarrow \frac{dy}{dx} = 4$	$\frac{dy}{dx} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{dy}{dx} = \frac{3x^2}{2y-3}$	A1ft	3

(b) final A1 $\sqrt{\ }$ **.** Note if the candidate inserts their x value and $y = 3$ into $\frac{dy}{dx} = \frac{3x^2}{2y-3}$, then an answer of $\frac{dy}{dx} =$ their x^2, ***may*** indicate a correct follow through.

[7]

5. (a) $3x^2 - y^2 + xy = 4$ (eqn *****)

$\underline{6x - 2y\frac{dy}{dx}} + \left(\underline{y + x\frac{dy}{dx}}\right) = \underline{0}$

$\left\{\frac{dy}{dx} = \frac{-6x-y}{x-2y}\right\}$ or $\left\{\frac{dy}{dx} = \frac{6x+y}{2y-x}\right\}$ *not necessarily required.*

$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x-y}{x-2y} = \frac{8}{3}$

giving $-18x - 3y = 8x - 16y$

giving $13y = 26x$

Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$

Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
Correct application $\underline{\underline{(\ )}}$ of product rule	B1
$(3x^2 - y^2) \rightarrow \left(\underline{6x - 2y\frac{dy}{dx}}\right)$ and $(4 \rightarrow \underline{0})$	$\underline{\text{A1}}$
Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.	M1*
Attempt to combine either terms in x or terms in y together	

		to give either ax or by.	dM1*	
		simplifying to give $\underline{y - 2x = 0}$ **AG**	A1 **cso**	6
	(b)	At P & Q, $y = 2x$. Substituting into eqn * gives $3x^2 - (2x)^2 + x(2x) = 4$ Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$ $y = 2x \Rightarrow y = \pm 4$ Hence coordinates are $\underline{(2, 4)}$ and $\underline{(-2, -4)}$		
		Attempt replacing y by $2x$ **in at least one** of the y terms in eqn *****	M1	
		Either $x = 2$ or $x = -2$	A1	
		Both (2, 4) and (–2, –4)	A1	3
				[9]
6.	(a)	$x^3 - 4y^2 = 12xy$ (eqn *****) $x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$		
		Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of = 0.	M1	
		$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$ $(y - 16)(y - 8) = 0$ $y = \frac{24 \pm \sqrt{576 - 4(128)}}{2}$ $y = 16$ or $y = 8$.		
		An attempt to solve the quadratic in y by either factorising or by the formula or by ***completing the square***.	dM1	
		Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1	3
	(b)	$3x^2 - 8y\frac{dy}{dx}; = \underline{\left(12y + 12x\frac{dy}{dx}\right)}$ $\left\{\frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y}\right\}$ @ (–8, 8), $\frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = \underline{-3}$,		

@ (–8, 16), $\frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = \underline{0}$.

Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$.

Ignore $\frac{dy}{dx} = ...$ M1

Correct LHS equation; A1;

Correct application of product rule (B1)

not necessarily required.

Substitutes x = –8 and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$. dM1

One gradient found. A1

Both gradients of $\underline{-3}$ and $\underline{0}$ ***correctly*** found. A1 **cso** 6

Aliter
Way 2

$$3x^2\frac{dy}{dx} - 8y; = \underline{\left(12y\frac{dx}{dy} + 12x\right)}$$

$$\left\{\frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y}\right\}$$

@ (–8, 8), $\frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = \underline{-3}$,

@ (–8, 16), $\frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = \underline{0}$.

Differentiates implicitly to include either $\pm kx^2\frac{dx}{dy}$ or $12y\frac{dx}{dy}$.

Ignore $\frac{dx}{dy} = ...$ M1

Correct LHS equation; A1;

Correct application of product rule (B1)

not necessarily required.

Substitutes x = –8 and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$ or $\frac{dx}{dy}$. dM1

One gradient found. A1

Both gradients of $\underline{-3}$ and $\underline{0}$ ***correctly*** found. A1 **cso** 6

Aliter
Way 3

$x^3 - 4y^2 = 12xy$ (eqn *)

$4y^2 + 12xy - x^3 = 0$

$$y = \frac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$$

$$y = \frac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$$

$$y = \frac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$$

$$y = -\frac{3}{2}x \pm \frac{1}{2}(9x^2 + x^3)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)(9x^2 + x^3)^{-\frac{1}{2}}; (18x + 3x^2)$$

$$\frac{dy}{dx} = -\frac{3}{2} \pm \frac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$$

$$@\ x = -8 \quad \frac{dy}{dx} = -\frac{3}{2}\frac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$$

$$= -\frac{3}{2} \pm \frac{48}{4\sqrt{(64)}} = -\frac{3}{2} \pm \frac{48}{32}$$

$$\therefore \frac{dy}{dx} = -\frac{3}{2} \pm \frac{3}{2} = \underline{-3}, \underline{0}.$$

A credible attempt to make y the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}(9x^2 + x^3)^{\frac{1}{2}}$.	M1	
$\frac{dy}{dx} = -\frac{3}{2} \pm k(9x^2 + x^3)^{-\frac{1}{2}}(g(x))$	A1	
$\frac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)(9x^2 + x^3)^{-\frac{1}{2}}; (18x + 3x^2)$	A1	
Substitutes $x = -8$ find any one of $\frac{dy}{dx}$.	dM1	
One gradient correctly found.	A1	
Both gradients of $\underline{-3}$ and $\underline{0}$ correctly found.	A1	6

[9]

7 (a) $\sin x + \cos y = 0.5$ (eqn *)

$\cos x - \sin y \dfrac{dy}{dx} = 0$ (eqn #)

$\dfrac{dy}{dx} = \underline{\dfrac{\cos x}{\sin y}}$

Differentiates implicitly to include $\pm \sin y \dfrac{dy}{dx}$. (Ignore $\left(\dfrac{dy}{dx} = \right)$.) M1

$\underline{\dfrac{\cos x}{\sin y}}$ A1 **cso** 2

(b) $\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$

giving $\underline{x = -\dfrac{\pi}{2}}$ or $\underline{x = \dfrac{\pi}{2}}$

When $x = -\dfrac{\pi}{2}, \sin\left(-\dfrac{\pi}{2}\right) + \cos y = 0.5$

When $x = \dfrac{\pi}{2}, \sin\left(\dfrac{\pi}{2}\right) + \cos y = 0.5$

$\Rightarrow \cos y = 1.5 \Rightarrow y$ has no solutions

$\Rightarrow \cos y = -0.5 \Rightarrow y = \dfrac{2\pi}{3}$ or $-\dfrac{2\pi}{3}$

In specified range $(x, y) = \underline{\left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)}$ and $\underline{\left(\dfrac{\pi}{2}, -\dfrac{2\pi}{3}\right)}$

Candidate realises that they need to solve 'their numerator' = 0
…or candidate sets $\dfrac{dy}{dx} = 0$ in their (eqn #) and attempts to solve
the resulting equation. M1ft

both $\underline{x = -\dfrac{\pi}{2}, \dfrac{\pi}{2}}$ or $\underline{x = \pm 90°}$ or awrt $\underline{x = \pm 1.57}$ required here A1

Substitutes either their $x = \dfrac{\pi}{2}$ or $x = -\dfrac{\pi}{2}$ into eqn ***** M1

Only one of $y = \underline{\dfrac{2\pi}{3}}$ or $\underline{-\dfrac{2\pi}{3}}$ or $\underline{120°}$
or $\underline{-120°}$ or awrt $\underline{-2.09}$ or awrt $\underline{2.09}$ A1

Only exact coordinates of $\underline{\left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)}$ and $\underline{\left(\dfrac{\pi}{2}, -\dfrac{2\pi}{3}\right)}$ A1

Do not award this mark if candidate states other coordinates inside the required range. A1 5

[7]

8. $\left\{\frac{\mathrm{d}y}{\mathrm{d}x}\right\} \times \; 6x - 4y\frac{\mathrm{d}y}{\mathrm{d}x} + 2 - 3\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ *or* $\pm 3\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$*.)* M1

Correct equation. A1

$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x+2}{4y+3}\right\}$

not necessarily required.

At (0, 1), $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{0+2}{4+3} = \frac{2}{7}$

Substituting x = 0 & y = 1 into an equation involving $\frac{\mathrm{d}y}{\mathrm{d}x}$; dM1

to give $\frac{2}{7}$ *or* $\frac{-2}{-7}$ A1 cso

Hence m(**N**) $= -\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$ A1ftoe.

*Uses m(T) to 'correctly' find m(**N**).*

Can be ft from "their tangent gradient".

Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$

or: **N**: $y = -\frac{7}{2}x + 1$

$y - 1 = m(x - 0)$ + *with 'their tangent or normal gradient';*

or

uses y = mx + 1 with 'their tangent or normal gradient' ; M1;

N: $7x + 2y - 2 = 0$

Correct equation in the form 'ax + by + c = 0', A1 oe

where a, b and c are integers. cso

[7]

Beware: $\frac{dy}{dx} = \frac{2}{7}$ does not necessarily imply the award of all the first four marks in this question.

So please ensure that you check candidates' initial differentiation before awarding the first A1 mark.

Beware: The final accuracy mark is for completely correct solutions.
If a candidate flukes the final line then they must be awarded A0.

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) = ∞, but obtains M0 if they write y – 1 = ∞(x – 0). If they write, however, **N**: x = 0, then can score M1.

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0,
and also obtains M1 if they write y – 1 = 0(x – 0)or y = 1.

Beware: The final **cso** refers to the whole question.

Aliter Way 2

$\left\{\frac{\not{d}\not{x}}{\not{d}\not{y}}\right\} \not{=}$ $6x\frac{dx}{dy} - 4y + 2\frac{dx}{dy} - 3 = 0$

Differentiates implicitly to include either $\pm kx\frac{dx}{dy}$ or $\pm 2\frac{dx}{dy}$ *(ignore* $\left(\frac{dx}{dy} = \right)$*.)* M1

Correct equation. A1

$\left\{\frac{dx}{dy} = \frac{4y+3}{6x+2}\right\}$

not necessarily required.

At (0, 1), $\frac{dx}{dy} = \frac{4+3}{0+2} = \frac{7}{2}$

Substituting x = 0 & y = 1 into an equation involving $\frac{dx}{dy}$*;* dM1

to give $\frac{7}{2}$ A1 cso

Hence m(**N**) = $\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$ A1ftoe.

*Uses m(**T**) or* $\frac{dx}{dy}$ *to 'correctly' find m(**N**).*

*Can be ft using "**–1** .* $\frac{dx}{dy}$ *".*

Either **N**: $y - 1 = -\frac{7}{2}(x-0)$ M1

or **N**: $y = -\frac{7}{2}x+1$

y – 1 = m(x – 0) with 'their tangent, $\frac{dx}{dy}$ *or normal gradient';*

or uses y = mx + 1 with 'their tangent, $\frac{dx}{dy}$ *or normal gradient';*

N: $7x + 2y - 2 = 0$

Correct equation in the form 'ax + by + c = 0', where a, b and c are integers. A1oe cso

[7]

Aliter Way 3

$2y^2 + 3y - 3x^2 - 2x - 5 = 0$

$\left(y+\frac{3}{4}\right)^2 - \frac{9}{16} = \frac{3x^2}{2} + x + \frac{5}{2}$

$y = \sqrt{\frac{3x^2}{2} + x + \frac{49}{16}} - \frac{3}{4}$

Differentiates using the chain rule; M1

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{3x^2}{2} + x + \frac{49}{16}\right)^{-\frac{1}{2}}(3x+1)$

Correct expression for $\frac{dy}{dx}$*;* A1 oe

At (0, 1),

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{49}{16}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{4}{7}\right) = \frac{2}{7}$$

Substituting x = 0 into an equation involving $\frac{dy}{dx}$; dM1

to give $\frac{2}{7}$ *or* $\frac{-2}{-7}$ A1 cso

Hence m(**N**) = $-\frac{7}{2}$ A1ft

*Uses m(**T**) to 'correctly' find m(**N**).*
Can be ft from "their tangent gradient".

Either **N**: $y - 1 \;\; -\frac{7}{2}(x - 0)$

or **N**: $y \;\; = -\frac{2}{7}x + 1$ M1

y – 1 = m(x – 0) with 'their tangent or normal gradient';
or uses y = mx + 1 with 'their tangent or normal gradient'

N: $7x + 2y - 2 = 0$ A1 oe

Correct equation in the form ' ax + by + c = 0',
where a, b and c are integers.

[7]

9. Differentiates

to obtain: $6x + 8y\frac{dy}{dx} \;\; - 2,$

.... $+ (6x \;\; \frac{dy}{dx} \;\; + 6y) = 0$ +(B1)

$$\left[\frac{dy}{dx} = \frac{2 - 6x - 6y}{6x + 8y}\right]$$

	Substitutes $x = 1$, $y = -2$ into expression involving $\frac{dy}{dx}$, to give $\frac{dy}{dx} = -\frac{8}{10}$	M1, A1	
	Uses line equation with numerical 'gradient' $y - (-2) =$ (their gradient) $(x - \mathbf{1})$ or finds c and uses $y =$ *(their gradient)* x + "c"	M1	
	To give $5y + 4x + 6 = 0$ (or equivalent = 0)	A1ft	
			[7]

10.	(a) Substitutes $t = 4$ to give V, = 1975.31 or 1975.30 or 1975 or 1980 (3 s.f)	M1, A1	2	
	(b) $\frac{dV}{dt} = -\ln 1.5 \times V$; = –800.92 or –800.9 or –801 *M1 needs ln 1.5 term*	M1 A1; A1	3	
	(c) rate of decrease in value on 1st January 2005	B1	1	
				[6]

11.	$2x + \left(2x\frac{dy}{dx} + 2y\right) - 6y\frac{dy}{dx} = 0$	M1 (A1) A1		
	$\frac{dy}{dx} = 0 \Rightarrow x + y = 0$ (or equivalent)	M1		
	Eliminating either variable and solving for at least one value of x or y. $y^2 - 2y^2 - 3y^2 + 16 = 0$ or the same equation in x	M1		
	$y = \pm 2$ or $x = \pm 2$	A1		
	$(2 - 2)$, $(-2, 2)$	A1	7	
	Note: $\frac{dy}{dx} = \frac{x + y}{3y - x}$			
	Alternative:			
	$3y^2 - 2xy - (x^2 + 16) = 0$			
	$y = \frac{2x \pm \sqrt{(16x^2 + 192)}}{6}$			
	$\frac{dy}{dx} = \frac{1}{3} \pm \frac{1}{3} . \frac{8x}{\sqrt{(16x^2 + 192)}}$	M1 A1 ± A1		
	$\frac{dy}{dx} = 0 \Rightarrow \frac{8x}{\sqrt{(16x^2 + 192)}} = \pm 1$	M1		
	$64x^2 = 16x^2 + 192$	M1 A1		
	$x = \pm 2$	A1	7	
	$(2, -2)$, $(-2, 2)$			
				[7]

12.	(a)	$10x, +(2y + 2x\frac{dy}{dx}), -6y\frac{dy}{dx} = 0$	M1, (B1), A1	
		At (1 , 2) $10 + (4 + 2\frac{dy}{dx}) - 12\frac{dy}{dx} = 0$	M1	
		$\therefore \frac{dy}{dx} = \frac{14}{10} = 1.4$ or $\frac{7}{5}$ or $1\frac{2}{5}$	A1	5
	(b)	The gradient of the normal is $-\frac{5}{7}$	M1	
		Its equation is $y - 2 = -\frac{5}{7}(x - 1)$	M1	
		(allow tangent)		
		$y = -\frac{5}{7}x + 2\frac{5}{7}$ or $y = -\frac{5}{7}x + \frac{19}{7}$	A1cao	3

[8]

13.	Differentiates w.r.t. x to give		
	$3x^2, -2x\frac{dy}{dx} + 2y, -4 + 3y^2\frac{dy}{dx} = 0$	M1, B1, A1	
	At (4, 3)		
	$48 - (8y' + 6) - 4 + 27y' = 0$	M1	
	$\Rightarrow y' = -\frac{38}{19} = -2$	A1	
	$\therefore$ Gradient of normal is $\frac{1}{2}$	M1	
	$\therefore y - 3 = \frac{1}{2}(x - 4)$	M1	
	i.e. $2y - 6 = x - 4$		
	$x - 2y + 2 = 0$	A1	8

[8]

1. This question was also well answered and the general principles of implicit differentiation were well understood. By far the commonest source of error was in differentiating 2^x; examples such as 2^x, $2^x \ln x$ and $x2^{x-1}$ were all regularly seen. Those who knew how to differentiate 2^x nearly always completed the question correctly, although a few had difficulty in finding $\frac{d}{dx}(2xy)$ correctly. A minority of candidates attempted the question by taking the logs of both sides of the printed equation or a rearrangement of the equation in the form $2^x = 2xy - y^2$. Correctly done, this leads to quite a neat solution, but, more frequently, errors, such as $\ln(2^x + y^2) = \ln 2^x + \ln y^2$, were seen.

It was noteworthy that a number of correct solutions were seen using partial differentiation, a topic which is not in the A level Mathematics or Further Mathematics specifications. These were, of course, awarded full marks.

2. As has been noted in earlier reports, the quality of work in the topic of implicit differentiation has improved in recent years and many candidates successfully differentiated the equation and rearranged it to find $\frac{dy}{dx}$. Some, however, forgot to differentiate the constant. A not infrequent, error was candidates writing $\frac{dy}{dx} = -2\sin 2x - 3\sin 3y\frac{dy}{dx}$ and then incorporating the superfluous $\frac{dy}{dx}$ on the left hand side of the equation into their answer. Errors like $\frac{dy}{dx}(\cos 3y)$ $= -\frac{1}{3}\sin 3y$.

were also seen. Part (b) was very well done. A few candidates gave the answer 20° , not recognising that the question required radians. Nearly all knew how to tackle part (c) although a few, as in Q2, spoilt otherwise completely correct solutions by not giving the answer in the form specified by the question.

3. As noted above work on this topic has shown a marked improvement and the median mark scored by candidates on this question was 8 out of 9. The only errors frequently seen were in differentiating ye^{-2x} implicitly with respect to x. A few candidates failed to read the question correctly and found the equation of the tangent instead of the normal or failed to give their answer to part (b) in the form requested.

4. A significant majority of candidates were able to score full marks on this question. In part (a), many candidates were able to differentiate implicitly and examiners noticed fewer candidates differentiating 8 incorrectly with respect to x to give 8. In part (b), many candidates were able to substitute $y = 3$ into C leading to the correct x-coordinate of –2. Several candidates either rearranged their C equation incorrectly to give $x = 2$ or had difficulty finding the cube root of –8. Some weaker candidates did not substitute $y = 3$ into C, but substituted $y = 3$ into the $\frac{dy}{dx}$ expression to give a gradient of x^2.

5. This question was generally well done with a majority of candidates scoring at least 6 of the 9 marks available.

In part (a), implicit differentiation was well handled with most candidates appreciating the need to apply the product rule to the *xy* term. A few candidates failed to differentiate the constant term and some wrote "$\frac{dy}{dx}$ = ..." before starting to differentiate the equation. After differentiating implicitly, the majority of candidates rearranged the resulting equation to make $\frac{dy}{dx}$ the subject before substituting $\frac{dy}{dx}$ as $\frac{8}{3}$ rather than substituting $\frac{8}{3}$ for $\frac{dy}{dx}$ in their differentiated equation. Many candidates were able to prove the result of $y - 2x = 0$. A surprising number of candidates when faced with manipulating the equation $\frac{6x + y}{2y - x} = \frac{8}{3}$, separated the fraction to incorrectly form two equations $6x + y = 8$ & $2y - x = 3$ and then proceeded to solve these equations simultaneously.

Some candidates, who were unsuccessful in completing part (a), gave up on the whole question even though it was still possible for them to obtain full marks in part (b). Other candidates, however, did not realise that they were expected to substitute $y = 2x$ into the equation of the curve and made no creditable progress with this part. Those candidates who used the substitution $y = 2x$ made fewer errors than those who used the substitution $x = \frac{y}{2}$. The most common errors in this part were for candidates to rewrite $-y^2$ as either $4x^2$ or $-2x^2$; or to solve the equation $x^2 = 4$ to give only $x = 2$ or even $x = \pm 4$. On finding $x = \pm 2$, some candidates went onto substitute these values back into the equation of the curve, forming a quadratic equation and usually finding "extra" unwanted points rather than simply doubling their two values of *x* to find the corresponding two values for *y*. Most candidates who progressed this far were able to link their values of *x* and *y* together, usually as coordinates.

6. This question was generally well done with many candidates scoring at least seven or eight of the nine marks available.

In part (a), the majority of candidates were able to use algebra to gain all three marks available with ease. It was disappointing, however, to see a significant minority of candidates at A2 level who were unable to correctly substitute $y = -8$ into the given equation or solve the resulting quadratic to find the correct values for *y*.

In part (b), implicit differentiation was well handled, with most candidates appreciating the need to apply the product rule to the $12xy$ term although errors in sign occurred particularly with those candidates who had initially rearranged the given equation so that all terms were on the LHS. A few candidates made errors in rearranging their correctly differentiated equation to make $\frac{dy}{dx}$ the subject. Also some candidates lost either one or two marks when manipulating their correctly substituted $\frac{dy}{dx}$ expressions to find the gradients.

7. In part (a), the majority of candidates were able to successfully differentiate the given equation to obtain a correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$, although there were a small proportion of candidates who appeared to "forget" to differentiate the constant term of 0.5. Some candidates, as was similar with Q3, produced a sign error when differentiating $\sin x$and $\cos y$ with respect to x. These candidates then went on to produce the correct answer for $\frac{\mathrm{d}y}{\mathrm{d}x}$, but lost the final accuracy mark. A few candidates incorrectly believed that the expression $\frac{\cos x}{\sin y}$ could be simplified to give $\cot x$.

In part (b), the majority of candidates realised that they needed to set their numerator equal to zero in order to solve $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Most candidates were then able to obtain at least one value for x, usually $x = \frac{\pi}{2}$, although $x = -\frac{\pi}{2}$ was not always found. A surprising number of candidates did not realise that they then needed to substitute their x value(s) back into the original equation in order for them to find y. Of those who did, little consideration was given to find all the solutions in the specified range, with a majority of these candidates finding $y = \frac{2\pi}{3}$, but only a minority of candidates also finding $y = -\frac{2\pi}{3}$. Therefore it was uncommon for candidates to score full marks in this part. Some candidates also incorrectly set their denominator equal to zero to find extra coordinates inside the range. Also another small minority of candidates stated other incorrect coordinates such as $\left(-\frac{\pi}{2}, \frac{2\pi}{3}\right)$ or $\left(-\frac{\pi}{2}, -\frac{2\pi}{3}\right)$ in addition to the two sets of coordinates required.
These candidates were penalised by losing the final accuracy mark.

8. This question was successfully completed by the majority of candidates. Whilst many demonstrated a good grasp of the idea of implicit differentiation there were a few who did not appear to know how to differentiate implicitly. Candidates who found an expression for $\frac{dy}{dx}$ in terms of x and y, before substituting in values of x= 1 and y = 1, were prone to errors in manipulation. Some candidates found the equation of the tangent and a number of candidates did not give the equation of the normal in the requested form.

9. `This question was generally well answered with most candidates showing good skills in differentiating explicitly. Candidates who found an expression for $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x and y, before substituting in values, were more prone to errors in manipulation. Some candidates found the equation of the normal and a number of candidates did not give the equation of the tangent in the requested form. It was quite common to see such statements as

$\frac{dx}{dy} = 6x + 8y\frac{dy}{dx} - 2 + \left(6x\frac{dy}{dx} + 6y\right) = 0$, but often subsequent correct working indicated that this was just poor presentation.

10. (a) Most understood the context of this problem and realised that they needed to use $t = 4$, although $t = 0$, 1 or 5 were often seen.

(b) Very few had any idea at all about how to differentiate V (many gave their answer as $-t(1.5)^{-t-1}$, or had a term $(1.5)^{-2t}$).

(c) The comments made in answer to the request to interpret their answer to part (b) were usually too generalised and vague. The examiners required a statement that the value of $\frac{dV}{dt}$ which had been found represented the rate of change of value on 1st January 2005

11. Almost all candidates could start this question and the majority could differentiate implicitly correctly. This is an area of work which has definitely improved in recent years. Many, having found $\frac{dy}{dx}$, could not use it and it was disturbing to find a substantial number of students in this relatively advanced A2 module proceeding from $\frac{x+y}{3y-x} = 0$ to $x + y = 3y - x$. Those who did obtain $y = -x$ often went no further but those who did could usually obtain both correct points, although extra incorrect points were often seen.

12. This was usually well done, but differentiation of a product caused problems for a number of candidates. Many still insisted on making $\frac{dy}{dx}$ the subject of their formula before substituting values for x and y. This often led to unnecessary algebraic errors.

13. No Report available for this question.