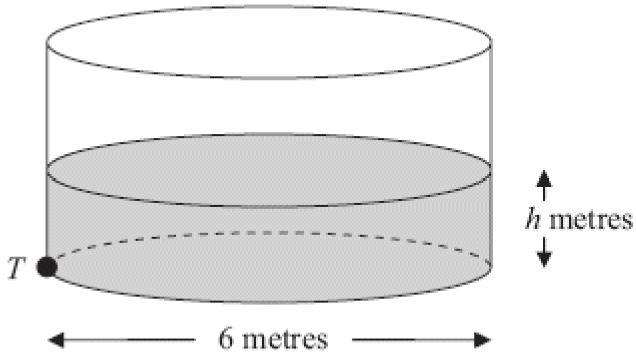
1.



The diagram above shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(a) Show that t minutes after the tap has been opened

$$75\frac{dh}{dt} = (4 - 5h)$$

(5)

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$

(6)
(Total 11 marks)

2. (a) Find $\int \frac{9x+6}{x}\,dx, x > 0.$

(2)

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$.

(6)
(Total 8 marks)

3. (a) Express $\frac{2}{4-y^2}$ in partial fractions.

(3)

(b) Hence obtain the solution of

$$2\cot x \frac{dy}{dx} = (4 - y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$.

(8)
(Total 11 marks)

4. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $cm^3\ s^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}\text{, where } k \text{ is a positive constant.}$$

(3)

When h = 25, water is leaking out of the hole at 400 $cm^3\ s^{-1}$.

(b) Show that k = 0.02

(1)

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh.$

(6)

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

(1)
(Total 13 marks)

5. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(a) solve the differential equation, giving P in terms of P_0, k and t.

(4)

Given also that $k = 2.5$,

(b) find the time taken, to the nearest minute, for the population to reach $2P_0$.

(3)

In an improved model the differential equation is given as

$$\frac{dP}{dt} = \lambda P \cos \lambda t,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(c) solve the second differential equation, giving P in terms of P_0, λ and t.

(4)

Given also that $\lambda = 2.5$,

(d) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model.

(3)
(Total 14 marks)

6. (a) Express $\dfrac{2x-1}{(x-1)(2x-3)}$ in partial fractions.

(3)

(b) Given that $x \geq 2$, find the general solution of the differential equation

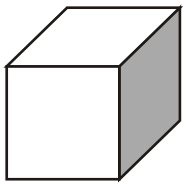
$$(2x-3)(x-1)\frac{dy}{dx} = (2x-1)y.$$

(5)

(c) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = f(x)$.

(4)
(Total 12 marks)

7.



At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^2 s^{-1}.

Show that

(a) $\frac{dx}{dt} = \frac{k}{x}$, where k is a constant to be found,

(4)

(b) $\frac{dV}{dt} = 2V^{\frac{1}{3}}$

(4)

Given that $V = 8$ when $t = 0$,

(c) solve the differential equation in part (b), and find the value of t when $V = 16\sqrt{2}$.

(7)
(Total 15 marks)

8. The rate of decrease of the concentration of a drug in the blood stream is proportional to the concentration C of the drug, which is present at that time. The time t is measured in hours from the administration of the drug and C is measured in micrograms per litre.

(a) Show that this process is described by the differential equation $\frac{dC}{dt} = -kC$,

explaining why k is a positive constant.

(1)

(b) Find the general solution of the differential equation, in the form $C = f(t)$.

(3)

After 4 hours, the concentration of the drug in the blood stream is reduced to 10% of its starting value C_0.

(c) Find the exact value of k.

(4)
(Total 8 marks)

9. The volume of a spherical balloon of radius r cm is V cm^3, where $V = \frac{4}{3}\pi r^3$.

(a) Find $\frac{dV}{dr}$

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(b) Using the chain rule, or otherwise, find an expression in terms of r and t for $\frac{dr}{dt}$.

(2)

(c) Given that $V = 0$ when $t = 0$, solve the differential equation $\frac{dV}{dt} = \frac{1000}{(2t+1)^2}$, to obtain V in terms of t.

(4)

(d) Hence, at time $t = 5$,

(i) find the radius of the balloon, giving your answer to 3 significant figures,

(3)

(ii) show that the rate of increase of the radius of the balloon is approximately 2.90×10^{-2} cm s^{-1}.

(2)
(Total 12 marks)

10. A spherical balloon is being inflated in such a way that the rate of increase of its volume, V cm^3, with respect to time t seconds is given by

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{k}{V}, \text{ where } k \text{ is a positive constant.}$$

Given that the radius of the balloon is r cm, and that $V = \frac{4}{3}\pi r^3$,

(a) prove that r satisfies the differential equation

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{B}{r^5}, \text{ where } B \text{ is a constant.}$$

(4)

(b) Find a general solution of the differential equation obtained in part (a).

(3)

When $t = 0$ the radius of the balloon is 5 cm, and when $t = 2$ the radius is 6 cm.

(c) Find the radius of the balloon when $t = 4$. Give your answer to 3 significant figures.

(5)
(Total 12 marks)

11. Liquid is pouring into a container at a constant rate of 20 cm^3 s^{-1} and is leaking out at a rate proportional to the volume of the liquid already in the container.

(a) Explain why, at time t seconds, the volume, V cm^3, of liquid in the container satisfies the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 20 - kV,$$

where k is a positive constant.

(2)

The container is initially empty.

(b) By solving the differential equation, show that

$$V = A + Be^{-kt},$$

giving the values of A and B in terms of k.

(6)

Given also that $\frac{dV}{dt} = 10$ when $t = 5$,

(c) find the volume of liquid in the container at 10 s after the start.

(5)
(Total 13 marks)

12. (a) Use integration by parts to show that

$$\int x \operatorname{cosec}^2\left(x + \frac{\pi}{6}\right)dx = -x \cot\left(x + \frac{\pi}{6}\right) + \ln\left[\sin\left(x + \frac{\pi}{6}\right)\right] + c, \qquad -\frac{\pi}{6} < x < \frac{\pi}{3}.$$

(3)

(b) Solve the differential equation

$$\sin^2\left(x + \frac{\pi}{6}\right)\frac{dy}{dx} = 2xy(y + 1)$$

to show that $\frac{1}{2}\ln\left|\frac{y}{y+1}\right| = -x\cot\left(x + \frac{\pi}{6}\right) + \ln\left[\sin\left(x + \frac{\pi}{6}\right)\right] + c.$

(6)

Given that $y = 1$ when $x = 0$,

(c) find the exact value of y when $x = \frac{\pi}{12}$.

(6)
(Total 15 marks)

13. A drop of oil is modelled as a circle of radius r. At time t

$$r = 4(1 - e^{-\lambda t}), \qquad t > 0,$$

where λ is a positive constant.

(a) Show that the area A of the circle satisfies

$$\frac{dA}{dt} = 32\pi\lambda\,(e^{-\lambda t} - e^{-2\lambda t}).$$

(5)

In an alternative model of the drop of oil its area A at time t satisfies

$$\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{t^2}, \qquad t > 0.$$

Given that the area of the drop is 1 at $t = 1$,

(b) find an expression for A in terms of t for this alternative model.

(7)

(c) Show that, in the alternative model, the value of A cannot exceed 4.

(1)
(Total 13 marks)

14. (a) Express $\dfrac{13 - 2x}{(2x - 3)(x + 1)}$ in partial fractions.

(4)

(b) Given that $y = 4$ at $x = 2$, use your answer to part (a) to find the solution of the differential equation

$$\frac{dy}{dx} = \frac{y(13 - 2x)}{(2x - 3)(x + 1)}, \quad x > 1.5$$

Express your answer in the form $y = f(x)$.

(7)
(Total 11 marks)

15. Fluid flows out of a cylindrical tank with constant cross section. At time t minutes, $t \geq 0$, the volume of fluid remaining in the tank is V m^3. The rate at which the fluid flows, in m^3 min^{-1}, is proportional to the square root of V.

(a) Show that the depth h metres of fluid in the tank satisfies the differential equation

$$\frac{dh}{dt} = -k\sqrt{h}, \qquad \text{where } k \text{ is a positive constant.}$$

(3)

(b) Show that the general solution of the differential equation may be written as

$$h = (A - Bt)^2, \qquad \text{where } A \text{ and } B \text{ are constants.}$$

(4)

Given that at time $t = 0$ the depth of fluid in the tank is 1 m, and that 5 minutes later the depth of fluid has reduced to 0.5 m,

(c) find the time, T minutes, which it takes for the tank to empty.

(3)

(d) Find the depth of water in the tank at time $0.5T$ minutes.

(2)

(Total 12 marks)

16. Liquid is poured into a container at a constant rate of 30 cm^3 s^{-1}. At time t seconds liquid is leaking from the container at a rate of $\frac{2}{15}V$ cm^3 s^{-1}, where V cm^3 is the volume of liquid in the container at that time.

(a) Show that

$$-15\frac{dV}{dt} = 2V - 450.$$

(3)

Given that $V = 1000$ when $t = 0$,

(b) find the solution of the differential equation, in the form $V = \text{f}(t)$.

(7)

(c) Find the limiting value of V as $t \to \infty$.

(1)

(Total 11 marks)

1. (a) $\frac{dV}{dt} = 0.48\pi - 0.6\pi h$ M1 A1

$V = 9\pi h \quad \Rightarrow \frac{dV}{dt} = 9\pi \frac{dh}{dt}$ B1

$9\pi \frac{dh}{dt} = 0.48\pi - 0.6\pi h$ M1

Leading to $75\frac{dh}{dt} = 4 - 5h$ ∗ cso A1 5

(b) $\int \frac{75}{4-5h} dh = \int 1 dt$ separating variables M1

$-15\ln(4-5h) = t \ (+C)$ M1 A1

$-15\ln(4-5h) = t + C$

When $t = 0$, $h = 0.2$

$-15\ln 3 = C$ M1

$t = 15\ln 3 - 15\ln(4-5h)$

When $h = 0.5$

$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4 M1 A1

Alternative for last 3 marks

$t = [-15\ln(4-5\text{h})]_{0.2}^{0.5}$

$= -15\ln 1.5 + 15\ln 3$ M1 M1

$= 15\ln\left(\frac{3}{1.5}\right) = 151\text{n } 2$ awrt 10.4 A1 6

[11]

2. (a) $\int \frac{9x+6}{x} dx = \int \left(9 + \frac{6}{x}\right) dx$ M1

$=9x + 6\ln x \ (+C)$ A1 2

(b) $\int \frac{1}{y^{\frac{1}{3}}}\mathrm{d}y = \int \frac{9x+6}{x}\mathrm{d}x$ Integral signs not necessary B1

$\int y^{-\frac{1}{3}}\mathrm{d}y = \int \frac{9x+6}{x}\mathrm{d}x$

$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x(+C)$ $\pm ky^{\frac{2}{3}} =$ their (a) M1

$\frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x(+C)$ ft their (a) A1ft

$y = 8$, $x = 1$

$\frac{3}{2}8^{\frac{2}{3}} = 9 + 6\ln 1 + C$ M1

$C = -3$ A1

$y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$

y2 $=(6x + 4\ln x - 2)^3$ $\left(= 8(3x + 2\ln x - 1)^3\right)$ A1 6

[8]

3. (a) $\frac{2}{4-y^2} \equiv \frac{2}{(2-y)(2+y)} \equiv \frac{A}{(2-y)} + \frac{B}{(2+y)}$

$2 \equiv A(2 + y) + B(2 - y)$

Let $y = -2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$

Let $y = 2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$

giving $\underline{\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}}$

Forming this identity.
NB: A & B are not assigned in this question M1

Either one of $A = \frac{1}{2}$ or $B = \frac{1}{2}$ A1

$\underline{\frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}}$, aef A1 **cao** 3

(If n**o working seen**, but candidate writes down ***correct partial fraction*** then award all three marks. If no working is seen but one of *A* or *B* is incorrect then M0A0A0.)

(b) $\int \frac{2}{4-y^2}\,\mathrm{d}y = \int \frac{1}{\cot x}\,\mathrm{d}x$

$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,\mathrm{d}x = \int \tan x\,\mathrm{d}x$

$\therefore -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) + (+c)$

$y = 0,\ x = \frac{\pi}{3} \Rightarrow -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$

$\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\}$

$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$

$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$

$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$

$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$

$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$

Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$

Separates variables as shown. Can be implied. Ignore the integral signs, and the ‘2’.	B1	
$\ln(\sec x)$ or $-\ln(\cos x)$	B1	
Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$	M1;	
their $\int \frac{1}{\cot x}\,\mathrm{d}x$ = LHS correct with ft for their A and B and no error with the “2” with or without $+c$	A1ft	
Use of $y = 0$ and $x = \frac{\pi}{3}$ in an integrated equation containing c;	M1*	
Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1	
Using the log laws correctly to obtain a single log term on both sides of the equation.	dM1*	
$\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 aef	8

[11]

4. (a) $\frac{\mathrm{d}V}{\mathrm{d}t} = 1600 - c\sqrt{h}$ or $\frac{\mathrm{d}V}{\mathrm{d}t} = 1600 - k\sqrt{h}$,

$(V = 4000\,h \Rightarrow)\ \frac{\mathrm{d}V}{\mathrm{d}h} = 4000$

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\frac{\mathrm{d}V}{\mathrm{d}t}}{\frac{\mathrm{d}V}{\mathrm{d}h}}$

Either, $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$

or $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$

Either of these statements	M1	
$\frac{\mathrm{d}V}{\mathrm{d}h} = 4000$ or $\frac{\mathrm{d}h}{\mathrm{d}V} = \frac{1}{4000}$	M1	
Convincing proof of $\frac{\mathrm{d}h}{\mathrm{d}t}$	A1 **AG**	3

(b) When $h = 25$ water ***leaks out such that*** $\frac{\mathrm{d}V}{\mathrm{d}t} = 400$

$400 = c\sqrt{h} \Rightarrow 400c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$

From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required

Proof that $k = 0.02$	B1 **AG**	1

Aliter
Way 2

$400 = 4000k\sqrt{h}$

$\Rightarrow 400 = 4000k\sqrt{25}$

$\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$

Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$	B1 **AG**	1

(c) $\frac{\mathrm{d}h}{\mathrm{d}t} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{\mathrm{d}h}{0.4 - k\sqrt{h}} = \int \mathrm{d}t$

$\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\mathrm{d}h \quad \frac{\div 0.02}{\div 0.02}$

time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\mathrm{d}h$

Separates the variables with $\int \frac{\mathrm{d}h}{0.4 - k\sqrt{h}}$ and $\int \mathrm{d}t$ on either side with integral signs not necessary. M1 oe

Correct proof A1 **AG** 2

(d) $\int_0^{100} \frac{50}{20-\sqrt{h}} \mathrm{d}h$ with substitution $h = (20 - x)^2$

$\frac{\mathrm{d}h}{\mathrm{d}x} = 2(20-x)(-1)$ or $\frac{\mathrm{d}h}{\mathrm{d}x} = -2(20-x)$

$h = (20-x)^2 \Rightarrow \sqrt{h} = 20 - x \Rightarrow x = 20 - \sqrt{h}$

$\int \frac{50}{20-\sqrt{h}} \mathrm{d}h = \int \frac{50}{x} . -2(20-x)\mathrm{d}x$

$\int = 100 \int \frac{x-20}{x} \mathrm{d}x$

$= 100 \int \left(1 - \frac{20}{x}\right) \mathrm{d}x$

$= 100(x - 20 \ln x)$ $(+ c)$

change limits: when $h = 0$ then $x = 20$
and when $h = 100$ then $x = 10$

$\int_0^{100} \frac{50}{20-\sqrt{h}} \mathrm{d}h = \left[100x - 2000 \ln x\right]_{20}^{10}$

or $\int_0^{100} \frac{50}{20-\sqrt{h}} \mathrm{d}h = \left[100(20-\sqrt{h}) - 2000 \ln(20-\sqrt{h})\right]_0^{100}$

$= (1000 - 2000 \ln 10) - (2000 - 2000 \ln 20)$
$= 2000 \ln 20 - 2000 \ln 10 - 1000$
$= 2000 \ln 2 - 1000$

Correct use of limits, ie. putting them in the correct way round
Either $x = 10$ and $x = 20$
or $h = 100$ and $h = 0$ ddM1

Combining logs to give...

$2000 \ln 2 - 1000$ or $-2000 \ln\left(\frac{1}{2}\right) - 1000$ A1 **aef** 6

(e) Time required = $2000 \ln 2 - 1000 = 386.2943611...$ sec
= 386 seconds (nearest second)
= 6 minutes and 26 seconds (nearest second)

<u>6 minutes, 26 seconds</u> B1 1

[13]

5. (a) $\frac{dP}{dt} = kP$ and $t = 0$, $P = P_0$ (1)

$\int \frac{dP}{P} = \int k\,dt$

$\ln P = kt$; $(+c)$

When $t = 0$, $P = P_0 \Rightarrow \ln P_0 = c$

(or $P = Ae^{kt} \Rightarrow P_0 = A$)

In $= P\ kt + \ln P_0 \Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$

Hence, $\underline{P = P_0 e^{kt}}$ 4

M1 Separates the variables with $\int \frac{dP}{P}$ and $\int k\,dt$ on either side with integral signs not necessary.

A1 Must see $\ln P$ and kt; Correct equation with/without $+ c$.

M1 Use of boundary condition (1) to attempt to find the constant of integration.

A1 $\underline{P = P_0 e^{kt}}$

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks.

Aliter
Way 2

$\frac{dP}{dt} = kP$ and $t = 0$, $P = P_0$ (1)

$\int \frac{dP}{kP} = \int 1\,dt$

$\frac{1}{k}\ln P = t; (+c)$

When $t = 0$, $P = P_0 \Rightarrow \frac{1}{k}\ln P_0 = c$

(or $P = Ae^{kt} \Rightarrow P_0 = A$)

$\frac{1}{k}\ln P = t + \frac{1}{k}\ln P_0 \Rightarrow \ln P = kt + \ln P_0$

$\Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$

Hence, $\underline{P = P_0 e^{kt}}$

M1 Separates the variables with $\int \frac{dP}{kP}$ and $\int dt$ on either side with integral signs not necessary.

A1 Must see $\frac{1}{k}\ln P$ and t; Correct equation with/without $+ c$.

M1 Use of boundary condition (1) to attempt to find the constant of integration.

A1 $\underline{P = P_0 e^{kt}}$

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks.

Aliter
Way 3

$\int \frac{dP}{kP} = \int 1 dt$

$\frac{1}{k}\ln(kP) = t; (+c)$

When $t = 0$, $P = P_0 \Rightarrow \frac{1}{k}\ln(kP_0) = c$

(or $kP = Ae^{kt} \Rightarrow kP_0 = A$)

$\frac{1}{k}\ln(kP) = t + \frac{1}{k}\ln(kP_0) \Rightarrow \ln(kP) = kt + \ln(kP_0)$

$\Rightarrow e^{\ln(kP)} = e^{kt+\ln(kP_0)} = e^{kt}.e^{\ln(kP_0)}$

$\Rightarrow kP = e^{kt}.(kP_0) \Rightarrow kP = kP_0 e^{kt}$

(or $kP = kP_0 e^{kt}$)

Hence, $\underline{P = P_0 e^{kt}}$

M1 Separates the variables with $\int \frac{dP}{kP}$ and $\int dt$ on either side with integral signs not necessary.

A1 Must see $\frac{1}{k}\ln(kP)$ and t; Correct equation with/without + c.

M1 Use of boundary condition (1) to attempt to find the constant of integration.

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks.

(b) $P = 2P_0$ & $k = 2.5 \Rightarrow \underline{2P_0 = P_0 e^{2.5t}}$

$e^{2.5t} = 2 \Rightarrow \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$

...or $e^{kt} = 2 \Rightarrow \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$

$\Rightarrow t = \frac{1}{2.5}\ln 2 = 0.277258872...$ days

$t = 0.277258872... \times 24 \times 60 = 399.252776...$ minutes

t = $\underline{399 \text{ min}}$ or t = $\underline{6 \text{ hr } 39 \text{ mins}}$ (to nearest minute) 3

M1 Substitutes $P = 2P_0$ into an expression involving P

M1 Eliminates P_0 and takes ln of both sides

A1 awrt t = $\underline{399}$ or $\underline{6 \text{ hr } 39 \text{ mins}}$

(c) $\frac{\mathrm{d}P}{\mathrm{d}t} = \lambda P \cos \lambda t$ and $t = 0, P = P_0$ (1)

$\int \frac{\mathrm{d}P}{P} = \int \lambda \cos \lambda t \mathrm{d}t$

$\ln P = \sin \lambda t; (+c)$

When $t = 0, P = P_0 \Rightarrow \ln P_0 = c$

(or $P = Ae^{\sin \lambda t} \Rightarrow P_0 = A$)

$\ln P = \sin \lambda t + \ln P_0 \Rightarrow e^{\ln P} = \mathrm{e}^{\sin \lambda t + \ln P_0} = \mathrm{e}^{\sin \lambda t}.\mathrm{e}^{\ln P_0}$

Hence, $\underline{P = P_0 e^{\sin \lambda t}}$ 4

M1 Separates the variables with $\int \frac{\mathrm{d}P}{P}$ and $\int \lambda \cos \lambda t \mathrm{d}t$ on either side with integral signs not necessary.

A1 Must see $\ln P$ and $\sin \lambda I$; Correct equation with/without + c.

M1 Use of boundary condition (1) to attempt to find the constant of integration.

A1 $\underline{P = P_0 e^{\sin kt}}$

$\underline{P = P_0 e^{\sin kt}}$ written down without the first M1 mark given scores all four marks.

Aliter
Way 2

$\frac{\mathrm{d}P}{\mathrm{d}t} = \lambda P \cos \lambda t$ and $t = 0, P = P_0$ (1)

$\int \frac{\mathrm{d}P}{\lambda P} = \int \lambda \cos \lambda t \mathrm{d}t$

$\frac{1}{\lambda} \ln P = \frac{1}{\lambda} \sin \lambda t; (+c)$

When $t = 0, P = P_0 \Rightarrow \frac{1}{\lambda} \ln P_0 = c$

(or $P = Ae^{\sin \lambda t} \Rightarrow P_0 = A$)

$\frac{1}{\lambda} \ln P = \frac{1}{\lambda} \sin \lambda t + \frac{1}{\lambda} \ln P_0 \Rightarrow \ln P = \sin \lambda t + \ln P_0$

$\Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t}.e^{\ln P_0}$

Hence, $\underline{P = P_0 e^{\sin \lambda t}}$

M1 Separates the variables with $\int \frac{\mathrm{d}P}{\lambda P}$ and $\int \cos \lambda t \mathrm{d}t$ on either side with integral signs not necessary.

A1 Must see $\frac{1}{\lambda} \ln P$ and $\frac{1}{\lambda} \sin \lambda t$; Correct equation with/without + c.

M1 Use of boundary condition (1) to attempt to find the constant of integration.

A1 $\underline{P = P_0 e^{\sin kt}}$

$\underline{P = P_0 e^{\sin kt}}$ written down without the first M1 mark given scores all four marks.

Aliter
Way 3

$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t = 0, P = P_0$ (1)

$\int \frac{dP}{\lambda P} = \int \lambda \cos \lambda t \, dt$

$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin \lambda t; (+c)$

When $t = 0, P = P_0 \Rightarrow \frac{1}{\lambda}\ln(\lambda P_0) = c$

(or $\lambda P = Ae^{\sin \lambda t} \Rightarrow \lambda P_0 = A$)

$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin \lambda t + \frac{1}{\lambda}\ln P_0$

$\Rightarrow \ln(\lambda P) = \sin \lambda t + \ln(\lambda P_0)$

$\Rightarrow e^{\ln(\lambda P)} = e^{\sin \lambda t + \ln(\lambda P_0)} = e^{\sin \lambda t}.e^{\ln(\lambda P_0)}$

$\Rightarrow \lambda P = e^{\sin \lambda t}.(\lambda P_0)$

(or $\lambda P = \lambda P_0 e^{\sin \lambda t}$)

Hence, $\underline{P = P_0 e^{\sin \lambda t}}$

M1 Separates the variables with $\int \frac{dP}{\lambda P}$ and $\int \cos \lambda t \, dt$ on either side with integral signs not necessary.

A1 Must see $\frac{1}{\lambda}\ln(\lambda P)$ and $\frac{1}{\lambda}\sin \lambda t$; Correct equation with/without + c.

M1 Use of boundary condition (1) to attempt to find the constant of integration.

A1 $\underline{P = P_0 e^{\sin kt}}$

(d) $P = 2P_0$ & λ 2.5 $\Rightarrow 2P_0 = P_0 e^{\sin 2.5t}$

$e^{\sin\ 2.5t} = 2 \Rightarrow \underline{\sin 2.5t = \ln 2}$

...or ... $e^{\lambda t} = 2 \Rightarrow \underline{\sin \lambda I = \ln 2}$

$\underline{t = \frac{1}{2.5}\sin^{-1}(\ln 2)}$

$t = 0.306338477...$

$t = 0.306338477... \times 24 \times 60 = 441.1274082...$ minutes

$t = \underline{441 \text{ min}}$ or $t = \underline{7 \text{ hr } 21 \text{ mins}}$ (to nearest minute) 3

M1 Eliminates P_0 and makes $\sin \lambda t$ or $\sin 2.5t$ the subject by taking ln's

dM1 Then rearranges to make t the subject. (must use $\sin^{-1}$)

A1 awrt t = <u>441</u> or <u>7 hr 21 mins</u>

[14]

6. (a) $\frac{2x-1}{(x-1)(2x-3)} \equiv \frac{A}{(x-1)} + \frac{B}{(2x-3)}$

$2x - 1 \equiv A(2x - 3) + B(x - 1)$

Forming this identity.
NB: A & B are not assigned in this question M1

Let $x = \frac{3}{2}$, $2 = B(\frac{1}{2}) \Rightarrow B = 4$

Let $x = 1$, $1 = A(-1) \Rightarrow A = -1$

giving $\frac{-1}{(x-1)} + \frac{4}{(2x-3)}$

either one of $A = -1$ or $B = 4$. A1
both correct for their A, B. A1 3

(b) & (c) $\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$

$= \int \frac{-1}{(x+1)} + \frac{4}{(2x-3)} dx$

$\therefore \ln y = -\ln(x - 1) + 2\ln(2x - 3) + c$

Separates variables as shown
Can be implied B1

Replaces RHS with their partial fraction to be integrated. M1ft

At least two terms in ln's M1
At least two ln terms correct A1ft
All three terms correct and '+ c' A1 5

$y = 10$, $x = 2$ gives $c = \ln 10$

$\therefore \ln y = -\ln(x - 1) + 2\ln(2x - 3) + \ln 10$

$\ln y = -\ln(x - 1) + \ln(2x - 3)^2 + \ln 10$

$\ln y = \ln\left(\frac{(2x-3)^2}{(x-1)}\right) + \ln 10$ or

$\ln y = \ln\left(\frac{10(2x-3)^2}{(x-1)}\right)$

$\underline{y = \frac{10(2x-3)^2}{(x-1)}}$

$c = \ln 10$ B1

Using the power law for logarithms	M1	
Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1	
$y = \dfrac{10(2x-3)^2}{(x-1)}$ or aef. isw	A1aef	4

Aliter
Way 2

$$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$$

Separates variables as shown Can be implied	B1

$$= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)} dx$$

Replaces RHS with their partial fraction to be integrated.	M1ft

$\therefore$ ln y = –ln(x – 1) + 2ln(2x – 3) + c

At least two terms in ln's	M1
At least two ln terms correct	A1ft
All three terms correct and '+ c'	A1
Decide to award B1 here!!!	B1

Note: The B1 mark (part (c)) should be awarded in the same place on ePEN as in the Way 1 approach.

$\therefore$ ln y = –ln(x – 1) + 2ln(2x – 3)^2 + c

Using the power law for logarithms	M1

$$\ln y = \ln\left(\frac{(2x-3)^2}{x-1}\right) + c$$

Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1

$$\ln y = \ln\left(\frac{A(2x-3)^2}{x-1}\right) \text{ where } c = \ln A$$

or $$e^{\ln y} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right)+c} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right)} e^c$$

$$y = \frac{A(2x-3)^2}{(x-1)}$$

y = 10, x = 2 gives A = 10

$$y = \frac{10(2x-3)^2}{(x-1)}$$

A = 10 for B1 award above

$y = \dfrac{10(2x-3)^2}{(x-1)}$ or aef & isw A1aef 5&4

Aliter
Way 3

$\int \dfrac{\mathrm{d}y}{y} = \int \dfrac{(2x-1)}{(2x-3)(x-1)}\mathrm{d}x$

$= \int \dfrac{-1}{(x+1)} + \dfrac{2}{(x-\frac{3}{2})}\,\mathrm{d}x$

$\therefore$ ln y = –ln(x – 1) + 2ln($x-\frac{3}{2}$) + c

Separates variables as shown
Can be implied B1

Replaces RHS with their partial fraction to be integrated. M1ft

At least two terms in ln's M1
At least two ln terms correct A1ft
All three terms correct and '+ c' A1 5

y = 10, x = 2 gives c = $\ln 10 - 2\ln\left(\frac{1}{2}\right) = \ln 40$

$\therefore$ ln y = –ln(x – 1) + 2ln($x-\frac{3}{2}$) + ln 40

ln y = –ln(x – 1) + ln($x-\frac{3}{2}$)^2 + ln 10

ln y = $\ln\left(\dfrac{(x-\frac{3}{2})^2}{(x-1)}\right) + \ln 40$ or

ln y = $\ln\left(\dfrac{40(x-\frac{3}{2})^2}{(x-1)}\right)$

$y = \dfrac{40(x-\frac{3}{2})^2}{(x-1)}$

c = ln 10 – 2 $\ln\left(\frac{1}{2}\right)$ or c = ln 40 B1 oe

Using the power law for logarithms M1

Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant *c*. M1

$y = \dfrac{40(x - \frac{3}{2})^2}{(x-1)}$ or aef. isw A1aef 4

Note: Please mark parts (b) and (c) together for any of the three ways.

[12]

7. (a) From question, $\dfrac{dS}{dt} = 8$ B1

$S = 6x^2 \Rightarrow \dfrac{dS}{dx} = 12x$ B1

$\dfrac{dx}{dt} = \dfrac{dS}{dt} \div \dfrac{dS}{dx} = \underline{\dfrac{8}{12x}}; = \dfrac{\frac{2}{3}}{x} \Rightarrow (k = \tfrac{2}{3})$

Candidate's $\dfrac{dS}{dt} \div \dfrac{dS}{dx}$; $\underline{\dfrac{8}{12x}}$ M1; <u>A1</u> oe 4

(b) $V = x^3 \Rightarrow \dfrac{dV}{dx} = 3x^2$ B1

$\dfrac{dV}{dt} = \dfrac{dV}{dx} \times \dfrac{dx}{dt} = 3x^2 . \left(\dfrac{2}{3x}\right); = 2x$

Candidate's $\dfrac{dV}{dx} \times \dfrac{dx}{dt}$; λx M1;A1ft

As $x = V^{\frac{1}{3}}$, then $\dfrac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG** A1

Use of $x = V^{\frac{1}{3}}$, *to give* $\dfrac{dV}{dt} = 2V^{\frac{1}{3}}$

(c) $\displaystyle\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\,dt$ B1

Separates the variables with $\displaystyle\int \frac{dV}{V^{\frac{1}{3}}}$ *or* $\displaystyle\int V^{-\frac{1}{3}}\,dV$ *on one side and* $\displaystyle\int 2\,dt$ *on the other side.*

integral signs not necessary.

$\displaystyle\int V^{-\frac{1}{3}}\,dV = \int 2\,dt$

$\tfrac{3}{2}V^{\frac{2}{3}} = 2t\,(+c)$

Attempts to integrate and ...
... must see $V^{\frac{2}{3}}$ *and 2t;* M1;
Correct equation with/without + c. A1

$\frac{3}{2}(8)^{\frac{2}{3}} = 2(0) + c \qquad \Rightarrow c = 6$

Use of V = 8 and t = 0 in a changed equation containing c ; c = 6 M1; A1

Hence: $\frac{3}{2}V^{\frac{2}{3}} = 2t + 6$

$\frac{3}{2}(16\sqrt{2})^{\frac{2}{3}} = 2t + 6 \quad \Rightarrow 12 = 2t + 6$ depM1

Having found their "c" candidate ...
... substitutes V = $16\sqrt{2}$ *into an equation involving V, t and "c".*

giving t = 3 A1cao 7

[15]

AliterWay 2

(b) $x = V^{\frac{1}{3}}$ & $S = 6x^2 \;\Rightarrow\; S = 6V^{\frac{2}{3}}$ B1ft

$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$ B1

$\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS} = 8.\left(\frac{1}{4V^{\frac{1}{3}}}\right); = \frac{2}{V^{-\frac{1}{3}}} = 2V^{\frac{1}{3}}$ **AG** M1; A1 4

Candidate's $\frac{dS}{dt} \times \frac{dV}{dS}; 2V^{\frac{1}{3}}$

Aliter Way 2

(c) $\int \frac{dV}{2V^{\frac{1}{3}}} = \int 1\,dt$ B1

Separates the variables with $\int \frac{dV}{2V^{\frac{1}{3}}}$ *or* $\int \frac{1}{2}V^{-\frac{1}{3}}\,dV$ *oe on one side and* $\int 1\,dt$ *on the other side.*
integral signs not necessary.

$\int \frac{1}{2}V^{-\frac{1}{3}}\,dV = \int 1\,dt$

$\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)V^{\frac{2}{3}} = t \;\; (+c)$

Attempts to integrate and ...
... must see $V^{\frac{2}{3}}$ *and t;* M1;

Correct equation with / without + c. A1

$\frac{3}{4}(8)^{\frac{2}{3}} = (0) + c \Rightarrow c = 3$

Use of V = 8 and t = 0 in a changed equation M1*;
containing c; c = 3 A1

Hence: $\frac{3}{4}V^{\frac{2}{3}} = t + 3$

$\frac{3}{4}(16\sqrt{2})^{\frac{2}{3}} = t + 3 \quad \Rightarrow 6 = t + 3$ depM1 *

Having found their "c" candidate ...
... substitutes V = $16\sqrt{2}$ *into an equation involving V, t and "c".*

giving $t = 3$. A1 cao 7

Aliter Way 3

(b) similar to way 1.

$V = x^3 \quad \Rightarrow \quad \frac{dV}{dx} = 3x^2$ B1

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS} = 3x^2 .8.\left(\frac{1}{12x}\right); = 2x$

Candidate's $\frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS}; \lambda x$ M1;A1ft

As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG** A1 4

Use of $x = V^{\frac{1}{3}}$*, to give* $\frac{dV}{dt} = 2V^{\frac{1}{3}}$

Aliter Way 3

(c) $\int \frac{dV}{2V^{\frac{1}{3}}} = \int 2\,dt$ B1

Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ *or* $\int V^{-\frac{1}{3}}\,dV$ *oe on one side and* $\int 2\,dt$ *on the other side.*
integral signs not necessary.

$\int V^{-\frac{1}{3}}\,\mathrm{d}V = \int 2\,\mathrm{d}t$

$V^{\frac{2}{3}} = \frac{4}{3}t \ \ (+c)$

Attempts to integrate and ...

... must see $V^{\frac{2}{3}}$ *and* $\frac{3}{4}\ t$; M1;

Correct equation with / without + c. A1

$(8)^{\frac{2}{3}} = \frac{4}{3}(0) + c \Rightarrow c = 4$

Use of V = 8 and t = 0 in a changed equation containing c; c = 4 M1;A1

Hence: $V^{\frac{2}{3}} = \frac{4}{3}t + 4$

$(16\sqrt{2})^{\frac{2}{3}} = \frac{4}{3}t + 6 \quad \Rightarrow 8 = \frac{4}{3}t + 4$ depM1

Having found their "c" candidate ...

... substitutes $V = 16\sqrt{2}$ *into an equation involving V, t and "c".*

giving t = 3. A1 cao 7

8. (a) $\frac{dC}{dt} = -kC$; rate of decrease/negative sign;

k constant of proportionality/positive constant B1 1

(b) $\int \frac{dC}{C} = -k\int \mathrm{d}t$ M1

$\therefore \ln C = -kt + \ln A$ M1

$\therefore\ C = Ae^{-kt}$ A1 3

(c) At $t = 0$ $C = C_0$, $\therefore A = C_0$ B1

and at $t = 4$ $C = \frac{1}{10}C_0$, $\therefore \frac{1}{10}C_0 = C_0e^{-4k}$, B1

$\therefore\ \frac{1}{10} = e^{-4t}$ and $\therefore -4k = \ln\frac{1}{10}$, $\therefore\ k = \frac{1}{4}\ln 10$ M1, A1 4

[8]

9.	(a)	$\frac{dV}{dr} = 4\pi r^2$	B1	1
	(b)	Uses $\frac{dr}{dt} = \frac{dV}{dt} \cdot \frac{dr}{dV}$ in any form, $= \frac{1000}{4\pi r^2 (2t+1)^2}$	M1, A1	2
	(c)	$V = \int 1000\,(2t+1)^{-2}\,dt$ and integrate to $p(2t+1)^{-1}$,		
		$= -500(2t+1)^{-1}\ (+c)$	M1, A1	
		Using V = 0 when t = 0 to find c , (c = 500, or equivalent)	M1	
		$\therefore V = 500(1 - \frac{1}{2t+1})$ (any form)	A1	4
	(d)	(i) Substitute t = 5 to give V,		
		then use $r = \sqrt[3]{\left(\frac{3V}{4\pi}\right)}$ to give r , = *4.77*	M1, A1	3
		(ii) Substitutes t = 5 and r = 'their value' into 'their' part (b)	M1	
		$\frac{dr}{dt} = 0.0289$ ($\approx 2.90 \times 10^{-2}$) (cm/s) AG	A1	2

[12]

10.	(a)	As $V = \frac{4}{3}\pi r^3$, then $\frac{dV}{dr} = 4\pi r^2$	M1	
		Using chain rule $\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr}$; $= \frac{k}{\frac{4}{3}\pi r^3} \times \frac{1}{4\pi r^2}$	M1 A1	
		$= \frac{B}{r^5}$ (*)	A1	4
	(b)	$\int r^5\,dr = \int B\,dt$	B1	
		$\therefore \frac{r^6}{6} = Bt + c$ (allow mark at this stage, does not need r =)	M1 A1	3

	(c)	Use $r = 5$ at $t = 0$ to give $c = \frac{5^6}{6}$ or 2604 or 2600	M1	
		Use $r = 6$ at $t = 2$ to give B = $\frac{6^5}{2} - \frac{5^6}{12}$ or 2586 or 2588 or 2590	M1	
		Put $t = 4$ to obtain r^6 (approx 78000)	M1 A1	
		Then take sixth root to obtain $r = 6.53$ (cm)	A1	5

[12]

11.	(a)	$\frac{dV}{dt}$ is the rate of increase of volume (with respect to time)	B1	
		$-kV$: k is constant of proportionality and the negative shows decrease (or loss) giving $\frac{dV}{dt} = 20 - kV$ (*)	B1	2
		These Bs are to be awarded independently		
	(b)	$\int \frac{1}{20 - kV} dV = \int 1 dt$	M1	
		separating variables		
		$-\frac{1}{k}\ln(20 - kV) = t\ (+C)$	M1 A1	
		Using $V = 0$, $t = 0$ to evaluate the constant of integration	M1	
		$c = -\frac{1}{k}\ln 20$		
		$t = \frac{1}{k}\ln\left(\frac{20}{20 - kV}\right)$		
		Obtaining answer in the form $V = A + B\,e^{-kt}$	M1	
		$V = \frac{20}{k} - \frac{20}{k}e^{-kt}$	A1	6
		Accept $\frac{20}{k}(1 - e^{-kt})$		
		Alternative to (b)		
		Using printed answer and differentiating $\frac{dV}{dt} = -kB\,e^{-kt}$	M1	
		Substituting into differential equation $-kB\,e^{-kt} = 20 - kA - kB\,e^{-kt}$	M1	
		$A = \frac{20}{k}$	M1 A1	
		Using $V = 0$, $t = 0$ in printed answer to obtain $A + B = 0$	M1	
		$B = -\frac{20}{k}$	A1	6

	(c)	$\frac{dV}{dt} = 10,\ t = 5 \Rightarrow 10 = 20e^{-kt} \Rightarrow k = \frac{1}{5}\ln 2 \approx 0.139$	M1 A1	
		At $t = 10,\ V = \frac{75}{\ln 2}$	M1 A1	5
		awrt 108		

[13]

12.	(a)	$I = \int x\ \text{cosec}^2(x + \frac{\pi}{6})dx = \int x d(-\cot(x + \frac{\pi}{6}))$	M1	
		$= -x\cot(x + \frac{\pi}{6}) + \int \cot(x + \frac{\pi}{6})dx$	A1	
		$= \underline{-x\cot(x+\frac{\pi}{6}) + \ln(\sin(x+\frac{\pi}{6})/c}$ (*)	A1c.s.o.	3
	(b)	$\int \frac{1}{y(1+y)}dx = \int 2x\text{cosec}^2(x+\frac{\pi}{6})dx$	M1	
		LHS = $\int \left(\frac{1}{y} - \frac{1}{1+y}\right)dy$	M1 A1	
		$\therefore \ln y - \ln\|1+y\|$ or $\ln\left\|\frac{y}{1+y}\right\| = 2(a)$	M1 M1	
		$\underline{\therefore \frac{1}{2}\ln\left\|\frac{y}{1+y}\right\| = -x\cot\left(x+\frac{\pi}{6}\right) + \ln\left\|\sin\left(x+\frac{\pi}{6}\right)\right\| + c}$ (*)	A1c.s.o.	6
	(c)	$y = 1, x = 0 \Rightarrow \frac{1}{2}\ln\frac{1}{2} = \ln\left(\sin\frac{\pi}{6}\right) + c$	M1	
		$\therefore c = -\frac{1}{2}\ln\frac{1}{2}$	A1 M1	
		$x = \frac{\pi}{12} \Rightarrow \frac{1}{2}\ln\left\|\frac{y}{1+y}\right\| = -\frac{\pi}{12}.1 + \ln\frac{1}{\sqrt{2}} - \frac{1}{2}\ln\frac{1}{2}$	A1	
		$\sqrt{\text{"}c\text{"}}$		
		(i.e. $\ln\left\|\frac{y}{1+y}\right\| = -\frac{\pi}{6}$)		
		$\frac{y+1}{y} = e^{\frac{\pi}{6}}$	M1	
		(o.e.)		
		$1 = y(e^{\frac{\pi}{6}} - 1)$		

$\therefore y = \dfrac{1}{e^{\frac{\pi}{6}} - 1}$ A1 6

(o.e.)

[15]

13. (a) $A = \pi r^2$, $\dfrac{dr}{dt} = 4\lambda e^{-\lambda t}$ B1, B1

$\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$, $\Rightarrow \dfrac{dA}{dt} = 2\pi \times 4(1 - e^{-\lambda t}) \times 4\lambda e^{-\lambda t}$ M1, M1

$\dfrac{dA}{dt} = 32\pi\lambda(e^{-\lambda t} - e^{-2\lambda t})$ A1cso 5

(b) $\int A^{-\frac{3}{2}} dA = \int t^{-2} dt$ M1

Separation

$\dfrac{A^{-\frac{1}{2}}}{-\frac{1}{2}} = \dfrac{t^{-1}}{-1}$ $(+ c)$ M1, A1

$-2 = -1 + c$ Use of (1, 1) M1

$c = -1$ A1

So $2A^{-\frac{1}{2}} = \dfrac{1}{t} + 1 \Rightarrow \sqrt{A} = \dfrac{2t}{1+t}$ Attempt $\sqrt{A}$ = or A = M1

i.e. $A = \dfrac{4t^2}{(1+t)^2}$ (or equivalent) A1 7

(c) Because $\dfrac{t^2}{(1+t)^2} < 1$ or $t^2 < (1 + t)^2$ $(\Rightarrow A < 4)$ B1 1

[13]

14. (a) Uses $\dfrac{A}{(2x-3)} + \dfrac{B}{(x+1)}$ M1

Considers $-2x + 13 = A(x + 1) + B(2x - 3)$ and substitutes $x = -1$ M1

or $x = 1.5$, or compares coefficients and solves simultaneous equations

To obtain A = 4 and B = –3. A1, A1 4

(b) Separates variables $\int \dfrac{1}{y} dy = \int \dfrac{4}{2x-3} - \dfrac{3}{x+1} dx$ M1

$\ln y = 2\ln(2x - 3) - 3\ln(x + 1) + C$ A1, B1 ft

Substitutes to give $\ln 4 = 2\ln 1 - 3\ln 3 + C$ and finds C (ln 108) M1

$\ln y = \ln(2x - 3)^2 - \ln(x + 1)^3 (+ \ln 108)$ M1

$= \ln \dfrac{C(2x-3)^2}{(x+1)}$ A1

$\therefore y = \dfrac{108(2x-3)^2}{(x+1)^3}$ A1 cso 7

Or $y = e^{2\ln(2x-3)-3\ln(x+1)+\ln 108}$ special case M1 A2

[11]

15. (a) $\dfrac{dV}{dt} = \pm c\sqrt{V}$ or $\dfrac{dV}{dt} \propto \sqrt{V}$ M1

As $V = Ah$, $\dfrac{dV}{dh} = A$ or $V \propto h$ M1

$\therefore$ use chain rule to obtain $\dfrac{dh}{dt} = -\dfrac{c}{A}\sqrt{V} = \dfrac{-c}{\sqrt{A}}\sqrt{h} = -k\sqrt{h}$ A1 3

(b) $\int \dfrac{dh}{h} = -\int -k\,dt$ M1<

$2h^{\frac{1}{2}} = A - kt$ M1 A1

$h^{\frac{1}{2}} = \dfrac{A}{2} - \dfrac{kt}{2}$

$h = (A - Bt)^2$ (*) A1 4

(c) $t = 0, h = 1$: $A = 1$ B1

$t = 5, h = 0.5$: $0.5 = (1 - 5B)^2$

$B = \dfrac{(1-\sqrt{0.5})}{5}$ $B = 0.0586)$ B1

$h = 0, t = \dfrac{A}{B} = \dfrac{5}{1-\sqrt{0.5}} = 17.1$ min B1 3

(d) $h = \dfrac{A^2}{4} = 0.25$ m M1 A1 2

[12]

16. (a) $\dfrac{dV}{dt} = 30 - \dfrac{2}{15}V$ M1 A1

$\Rightarrow -15\dfrac{dV}{dt} = -450 + 2V$, no wrong working seen A1* 3

(b)	Separating the variables $\Rightarrow -\frac{15}{2V-450}\,\mathrm{d}V = \mathrm{d}t$	M1	
	Integrating to obtain $-\frac{15}{2}\ln\lvert 2V-450\rvert = t$ OR $-\frac{15}{2}\ln\lvert V-225\rvert = t$	dM1 A1	
	Using limits correctly or finding c $(-\frac{15}{2}\ln 1550$ *OR* $-\frac{15}{2}\ln 775)$	M1	
	$\ln\frac{2V-450}{1550} = -\frac{2}{15}t$, or equivalent	A1	
	Rearranging to give $V = 225 + 775e^{-\frac{2}{13}t}$.	dM1 A1	7
(c)	$V = 225$	B1	1

[11]

1. Many found part (a) difficult and it was quite common to see candidates leave a blank space here and proceed to solve part (b), often correctly. A satisfactory proof requires summarising the information given in the question in an equation, such as $\frac{dV}{dT} = 0.48\pi - 0.6\pi h$, but many could not do this or began with the incorrect $\frac{dh}{dt} = 0.48\pi - 0.6\pi h$. Some also found difficulty in obtaining a correct expression for the volume of water in the tank and there was some confusion as to which was the variable in expressions for the volume. Sometimes expressions of the form $V = \pi r^2 h$ were differentiated with respect to r, which in this question is a constant. If they started appropriately, nearly all candidates could use the chain rule correctly to complete the proof.

Part (b) was often well done and many fully correct solutions were seen. As noted in the introduction above, some poor algebra was seen in rearranging the equation but, if that was done correctly, candidates were nearly always able to demonstrate a complete method of solution although, as expected, slips were made in the sign and the constants when integrating. Very few candidates completed the question using definite integration. Most used a constant of integration (arbitrary constant) and showed that they knew how to evaluate it and use it to complete the question.

2. Part (a) of this question proved awkward for many. The integral can be carried out simply by decomposition, using techniques available in module C1. It was not unusual to see integration by parts attempted. This method will work if it is known how to integrate ln x , but this requires a further integration by parts and complicates the question unnecessarily. In part (b), most could separate the variables correctly but the integration of $\frac{1}{y^{\frac{1}{3}}}$, again a C1 topic, was frequently incorrect.

Weakness in algebra sometimes caused those who could otherwise complete the question to lose the last mark as they could not proceed from $y^{\frac{2}{3}} = 6x + 41n\,x - 2$ to $y^2 = (6x + 41\ln x - 2)^3$. Incorrect answers, such as $y^2 = 216x^3 + 64 \ln x^3 - 8$, were common in otherwise correct solutions.

3. In part (a), many candidates realised that they needed to factorise the denominator to give two linear factors, and usually proceeded to give a fully correct solution. A few candidates, however, thought that $4 - y^2$ was an example of a repeated linear factor and tried to split up their fraction up accordingly. Some candidates struggled with factorising $4 - y^2$ giving answers such as $(4 + y)(4 - y)$ or $(y + 2)(y - 2)$. The majority of candidates were able to write down the correct identity to find their constants, although a noticeable number of candidates, when solving $4A = 2$ found $A = 2$.

A significant minority of candidates who completed part (a) correctly made no attempt at part (b). About half of the candidates in part (b) were able to separate out the variables correctly. Many of these candidates spotted the link with part (a). It was pleasing that candidates who progressed this far were able to correctly integrate tan x and correctly find the two ln terms by integrating their partial fraction. Common errors at this point were integrating tan x to give $\sec^2 x$ and the sign error involved when integrating $\frac{K}{2-y}$. A significant number of candidates at this point did not attempt to find a constant of integration. Other candidates substituted $x=\frac{\pi}{3}$ and $y = 0$ into an integrated equation which did not contain a constant of integration. A majority of candidates who found the constant of integration struggled to simplify their equation down to an equation with a single ln term on each side. The most common error of these candidates was to believe that $\ln A + \ln B = \ln C$ implies $A + B = C$.

Of all the 8 questions, this was the most demanding in terms of a need for accuracy. Fewer than 10% of candidates were able to score all 11 marks in this question, although statistics show that about half of the candidates were able to score at least 5 marks.

4. This proved by far the most difficult question on the paper and discriminated well for those candidates who were above the grade *A* threshold for this paper. Only a few candidates were able to score above 8 or 9 marks on this question.

Many 'fudged' answers were seen in part (a). A more rigorous approach using the chain rule of $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ was required, with candidates being expected to state $\frac{dV}{dt}$ and $\frac{dV}{dh}$ (or its reciprocal). The constant of proportionality also proved to be a difficulty in this and the following part.

Few convincing proofs were seen in part (b) with a significant number of candidates not understanding how to represent 400 cm^3 s^{-1} algebraically.

Only a minority of candidates were able to correctly separate the variables in part (c). Far too often, expressions including $\int \frac{dh}{0.4} = \int 0.02\sqrt{h}\,dt$ were seen by examiners. There were a significant number of candidates who having written $\int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$ could not progress to the given answer by multiplying the integral on the left hand side by $\frac{50}{50}$.

Despite struggling with the previous three parts, a majority of candidates were able to attempt part (d), although only a few candidates were able to produce the correct final exact answer. A majority of candidates who attempted this part managed to correctly obtain $\frac{dh}{dx} = 2x - 40$ and then use this and the given substitution to write down an integral in x. At this point a significant number of candidates were unable to manipulate the expression $k\left(\frac{x-10}{x}\right)$ into an expression of the form $k\left(1-\frac{20}{x}\right)$. The converted limits $x = 10$ and $x = 20$,caused an added problem for those candidates who progressed further, with a significant number of candidates incorrectly applying $x = 10$ as their lower limit and $x = 20$ as their upper limit.

A time of 6 minutes 26 seconds was rarity in part (e).

5. Many candidates, who answered part (a), were able to separate the variables correctly and integrate both sides of their equation to obtain $\ln P = kt$. At this point a significant number of candidates either omitted the constant of integration or were unable to deal with the boundary conditions given in the question. Some candidates, for example, wrote down $P = e^{kt} + c$; and stated that $c = P_0$ to give the common incorrect solution of $P = P_0 + e^{kt}$. Other candidates used P_0 instead of P in their attempts, and then struggled to find the constant of integration. Some candidates, who correctly evaluated the constant of integration, did not make P the subject of the equation but left their answer as $\ln P = kt + \ln P_0$.

Those candidates who had successfully answered part (a) were able to gain most of the marks available in part (b). A few of these candidates, however, struggled to convert the correct time in hours to the correct time in minutes. Those who did not progress well in part (a) may have gained only a method mark in part (b) by replacing P in their part (a) equation with $2P_0$.

Those candidates who were successful in the first two parts of this question usually succeeded to score most of the marks available in parts (c) and (d). In part (c) some candidates incorrectly integrated $\lambda \cos \lambda t$. In part (d), a significant number of candidates found difficultly in solving the equation $\sin(2.5t) = \ln 2$. It was not uncommon for some of these candidates to write $t = \frac{\ln 2}{\sin(2.5)}$. Also, in part (d), some candidates did not work in radians when evaluating $t = \arcsin(\ln 2)$.

There were a significant minority of candidates who tackled this question with ease scoring at least thirteen of the fourteen marks available.

6. In part (a), nearly all candidates were able to form the required partial fractions accurately and efficiently. Most candidates substituted $x = \frac{3}{2}$ and $x = 1$ into the identity $2x - 1 \equiv A(2x - 3) + B(x - 1)$, but the cover-up rule and the method of equating coefficients were also used.

In part (b), some candidates failed to proceed much further, through not knowing how to separate the variables or not recognising that their solution to part (a) provided a clue as how to proceed to solve the differential equation.

Many candidates were able to write the general solution as an equation involving three logarithmic terms and a constant of integration 'c'. Some candidates, however, omitted this constant, whilst other candidates incorrectly integrated $\frac{4}{2x-1}$ to give either $4\ln|2x - 1|$ or $\ln|2x - 1|$.

In part (c), a majority of candidates realised that they were required to find the constant of integration and were able to evaluate this constant correctly. Only a minority of candidates, however, were able to use the laws of logarithms in the correct order to give their particular solution in the form $y = \mathrm{f}(x)$. There were a significant number of candidates who arrived at the incorrect particular solution of $y = \frac{(2x-3)^2}{(x-1)} + 9$.

7. A significant number of candidates found parts (a) and (b) difficult although other candidates answered these two parts of the question with ease. Those candidates who used $\frac{dx}{dt} = \frac{dx}{ds} \times \frac{ds}{dt}$ in part (a) and $^{dV}=^{dV}\times^{dx}$in part (b) managed better than those candidates who worked with V and S or V, S and x. The most common error in these parts was for candidates to incorrectly quote the surface area S, as x^2or $4x^2$instead of $6x^2$.
Part (c) was tackled better than the rest of the question with many candidates recognising the need to separate the variables, integrate, find the constant of integration and substitute for V. Many candidates were able to score full marks easily on this part. There was, nevertheless, plenty of scope for errors to occur at all stages in the solution. Those who separated out $\frac{1}{2V^{\frac{1}{3}}}$ frequently simplified this to $2V^{-\frac{1}{3}}$. After integration incorrect expressions such as $V^{\frac{4}{3}}$, $V^{\frac{1}{3}}$ and $\ln V^{\frac{1}{3}}$ all regularly appeared. A number of candidates did not use a constant of integration. Other candidates found difficultly in working with $\left(16\sqrt{2}\right)^{\frac{2}{3}}$.

8. Part (a) specifically asked the candidates to explain why *k* was a positive constant. Although candidates linked the negative sign with the rate of decrease, they often did not then explain how this related to *k*. When explanations were given they tended to be verbose.

Candidates who used C as a constant of integration in part (b) often confused themselves. It was not unusual to see no constant of integration or to see the correct statement ln $C = -kt + A$ leading to the incorrect statement $C = e^{-kt} + e^{A}$. It seemed that candidates were familiar with an exponential decay result of the form $C = Ae^{-kt}$ but were not necessarily sure how to get to this result from a given differential equation.

Errors in method were seen in part (c) when candidates had omitted the constant of integration in part (b) and full marks were only given to a full and correct solution. Many candidates ignored the given starting value of $C = C_0$ at time $t = 0$ and instead used, for example, $C_0 = 1$ or $C_0 = 100$. This was allowed provided the candidates went on to use $C = 0.1\times$their value of C_0 at time $t = 4$. The most frequent error was the use of $0.9C_0$ or its equivalent. This was another question where some candidates ignored the requirement for an exact value.

9. The fact that this question had so many parts, with a good degree of independence, did enable the majority of candidates to do quite well. All but the weakest candidates scored the first mark and the first 3 were gained by most. The integration in part (c) did cause problems: examples of the more usual mistakes were to write

$\int \frac{1}{(2t+1)^2}\,dt = -\frac{1}{2t+1}$ or $\frac{1}{2t+1}$ or $\frac{2}{2t+1}$ or $\frac{k}{(2t+1)^3}$, or to omit the constant of integration or assume it equal to zero; two of the mistakes which came more into the "howler" category were

$\int \frac{1}{1000}\,dV = \ln 1000V$ *or* $V\ln 1000$ and

$\int \frac{1}{(2t+1)^2}dt = \int \frac{1}{4t^2+4t+1}dt = \int \frac{1}{4t^2} + \frac{1}{4t} + \frac{1}{1}dt = \ldots\ldots\ .$

Many candidates were able to gain the method marks in parts (d) and (e).

10. This was well done on the whole but there were some slips simplifying the algebra in part (a) and a significant number of candidates persevered with complicated constants, involving π^2, throughout the question. The separation of variables and solution of the differential equation was answered well. In c) most candidates had some success trying to find their constants, but a number found the arithmetic difficult and did not obtain a correct answer.

11. This proved by far the most difficult question on the paper. When an explanation is asked for an equation, the candidate must make specific reference to the elements of that equation in their explanation. For example, $\frac{dV}{dt}$, needed to be explicitly identified with the rate of change (or increase) of volume with respect to time. The loss of liquid due to leakage at a rate proportional to the volume needed to be identified with the term $-kV$. Few could make any progress at all with part (b). The question was at a level of abstraction unexpected by most candidates and there was much confusion between the various constants, k, A and B, occurring in the question. After an attempt at part (a), it was not uncommon for candidates to simply give up. The majority were unable to separate the variables. When separation and integration were achieved the constant of integration, if it was recognised at all, tended to be confused with k or subsumed into A or B. Those who got an answer in the correct form in part (b), even if it was incorrect, could usually demonstrate a correct method in part (c) and obtain some credit.

12. Candidates found this question challenging; however those who read all the demands of the question carefully were able to score some marks, whilst quite an appreciable minority scored them all. In part (a), the crucial step involved keeping signs under control. Seeing

$$-x\cot\left(x+\frac{x}{6}\right)-\int -\cot\left(x+\frac{x}{6}\right)dx$$

or a correct equivalent, demonstrated to examiners a clear method. Sign confusions sometimes led to a solution differing from the printed answer.

Part (b) was the main source for the loss of marks in this question. It was disappointing that so many candidates rushed through with barely more than three lines of working between separating the variables and quoting the printed answer, losing the opportunity to demonstrate their skills in methods of integration. The majority separated the variables correctly. Very few made any attempt to include the critical partial fractions step, merely stating

$$\frac{1}{2}\int\frac{1}{y(y+1)}dy=\frac{1}{2}\ln\left(\frac{y}{1+y}\right)$$

as printed. Some did not recognise the right hand side of their integral related to part (a), producing copious amounts of working leading to nowhere.

In part (c), the working to evaluate the constant c was often untidy and careless. Those who persevered to a stage of the form $\ln P = Q + R$ generally were unable to move on to $P = e^{Q+R}$ in a satisfactory manner, often writing $P = e^{Q} + e^{R}$.

13. There were two common approaches used in part (a); substituting for r to obtain a formula for A in terms of t or using the chain rule. The inevitable errors involving signs and λ were seen with both methods and the examiners were disappointed that some candidates did not seem to know the formula for the area of a circle: $2\pi r, \frac{1}{2}\pi r^2$ and $4\pi r^2$ were common mistakes. Part (b) proved more testing. Most could separate the variables but the integration of negative powers caused problems for some who tried to use the ln function. Many did solve the differential equation successfully though sometimes they ran into difficulties by trying to make A the subject before finding the value of their arbitrary constant. The final two marks were only scored by the algebraically dexterous. There was some poor work here and seeing $\frac{2}{\sqrt{A}} = \frac{1}{t} + 1$ followed by $\frac{4}{A} = \frac{1}{t^2} + 1$ or $\frac{\sqrt{A}}{2} = t + 1$ was not uncommon. The final part eluded most. Those who had a correct answer to part (b) sometimes looked at the effect on A of $t \to \infty$ but only a small minority argued that since $t > 0$ then $t^2 < (1+t)^2$, and therefore $A < 4$.

14. The partial fractions were found easily by most of the candidates, with very few errors. Some had difficulty separating the variables, but it was still possible to continue with the answer and to obtain some credit. The log integration was performed well this time and the majority of difficulties were finding the correct constant and using it correctly in conjunction with combining the logs.

15. The first part of this proved to be particularly troublesome. M1 was given quite often for $\frac{dV}{dt} = kV$, and a number recognised that the constant cross section implied that V was proportional to r. Many candidates had elements of the correct method, but they failed to link these together to form a convincing argument. Several candidates however were not able to make use of the information about the “constant cross section”, or to interpret the rate and the proportionality.

In part (b) most candidates did try to separate the variables. Integrating $1/\sqrt{h}$ caused problems, with many candidates giving an answer involving $\ln\sqrt{h}$, and with better attempts losing the factor 2. The constant of integration was often missing. Several candidates failed to make a convincing link between their answer and the form given on the question paper.

The standard of algebra and arithmetic in the last two parts was disappointing. Most candidates did make some attempt to substitute the given values, but few reached correct answers. Many found $A = 1$, then when finding B expanded the expression into a quadratic which then they tried to solve using the formula. Others square rooted both sides instead of squaring to remove the square root. Many candidates substituted 0.5 rather than $0.5T$ in part (d), indicating lack of comprehension of the question.

16. This was found to be the most difficult question on the paper. Setting up the differential equation in part (a) was usually answered well, as the answer was printed. The separation of variables proved difficult and this meant that part (b) was virtually inaccessible for many candidates. Some of those who correctly separated the variables had difficulties with signs, having presumably not noticed the boundary conditions. Another group did not deal correctly with the constant of integration, leaving it as part of the exponential. It was possible to obtain the answer to part (c) from the original question, even if part (b) had proved impossible. Some (Further Mathematics) candidates did use the integrating factor method to answer part (b).