Exercise 7H

1 $OP = \sqrt{2^2 + 8^2 + (-4)^2}$
$= \sqrt{4 + 64 + 16} = \sqrt{84}$
$= 2\sqrt{21} \approx 9.17$ (3 s.f.)

2 $OP = \sqrt{7^2 + 7^2 + 7^2}$
$= \sqrt{49 + 49 + 49} = \sqrt{147}$
$= 7\sqrt{3} \approx 12.1$ (3 s.f.)

3 a $AB = \sqrt{(3-1)^2 + (0-(-1))^2 + (5-8)^2}$
$= \sqrt{2^2 + 1^2 + (-3)^2}$
$= \sqrt{14} \approx 3.74$ (3 s.f.)

b $AB = \sqrt{(8-(-3))^2 + (11-1)^2 + (8-6)^2}$
$= \sqrt{11^2 + 10^2 + 2^2}$
$= \sqrt{225} = 15$

c $AB = \sqrt{(3-3)^2 + (5-10)^2 + (-2-3)^2}$
$= \sqrt{0^2 + (-5)^2 + (-5)^2}$
$= \sqrt{50} = 5\sqrt{2} \approx 7.07$ (3 s.f.)

d $AB = \sqrt{(-1-4)^2 + (-2-(-1))^2 + (5-3)^2}$
$= \sqrt{(-5)^2 + (-1)^2 + 2^2}$
$= \sqrt{30} \approx 5.48$ (3 s.f.)

4 $AB = \sqrt{(7-k)^2 + (-1-0)^2 + (2-4)^2} = 3$
$\sqrt{(49 - 14k + k^2) + 1 + 4} = 3$
$49 - 14k + k^2 + 1 + 4 = 9$
$k^2 - 14k + 45 = 0$
$(k-5)(k-9) = 0$
$k = 5$ or $k = 9$

5 $AB = \sqrt{(5-1)^2 + (3-k)^2 + (-8-(-3))^2}$
$= 3\sqrt{10}$
$\sqrt{16 + (9 - 6k + k^2) + 25} = 3\sqrt{10}$
$16 + 9 - 6k + k^2 + 25 = 9 \times 10$
$k^2 - 6k - 40 = 0$
$(k+4)(k-10) = 0$
$k = -4$ or $k = 10$

Challenge

a Coordinates of other points in the plane
$x = 1$ will be (1, −3, 4) and (1, −3, −2).

Coordinates of other points in the plane
$x = 7$ will be (7, 3, 4), (7, 3, −2) and
(7, −3, −2).

b Shortest route for the ant will be from A to
half way along one of the opposite edges
and then across the next face to C.

Distance $= 2 \times \sqrt{6^2 + 3^2} = 2 \times \sqrt{45}$
$= 2 \times 3\sqrt{5} = 6\sqrt{5}$