Exercise 7G

1 a i $\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = 4\mathbf{i} + 5\mathbf{j}$,
$\overrightarrow{OC} = -2\mathbf{i} + 6\mathbf{j}$

ii $\overrightarrow{AB} = B - A$
$= (4\mathbf{i} + 5\mathbf{j}) - (3\mathbf{i} - \mathbf{j})$
$= 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{i} + \mathbf{j}$
$= \mathbf{i} + 6\mathbf{j}$

iii $\overrightarrow{AC} = C - A$
$= (-2\mathbf{i} + 6\mathbf{j}) - (3\mathbf{i} - \mathbf{j})$
$= -2\mathbf{i} + 6\mathbf{j} - 3\mathbf{i} + \mathbf{j}$
$= -5\mathbf{i} + 7\mathbf{j}$

b i $|\overrightarrow{OC}| = |-2\mathbf{i} + 6\mathbf{j}|$
$= \sqrt{(-2)^2 + 6^2}$
$= \sqrt{40}$
$= \sqrt{4}\sqrt{10}$
$= 2\sqrt{10}$

ii $|\overrightarrow{AB}| = |\mathbf{i} + 6\mathbf{j}|$
$= \sqrt{1^2 + 6^2}$
$= \sqrt{37}$

iii $|\overrightarrow{AC}| = |-5\mathbf{i} + 7\mathbf{j}|$
$= \sqrt{(-5)^2 + 7^2}$
$= \sqrt{74}$

2 a $\overrightarrow{PQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$
$= -(4\mathbf{i} - 3\mathbf{j}) + 3\mathbf{i} + 2\mathbf{j}$
$= -\mathbf{i} + 5\mathbf{j}$
or
$-\begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

b i $|\overrightarrow{OP}| = \sqrt{4^2 + (-3)^2}$
$= \sqrt{25} = 5$

ii $|\overrightarrow{OQ}| = \sqrt{3^2 + 2^2}$
$= \sqrt{13}$

2 b iii $|\overrightarrow{PQ}| = \sqrt{(-1)^2 + 5^2}$
$= \sqrt{26}$

3 a $\overrightarrow{PQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$
$5\mathbf{i} + 6\mathbf{j} = -\overrightarrow{OP} + (4\mathbf{i} - 3\mathbf{j})$

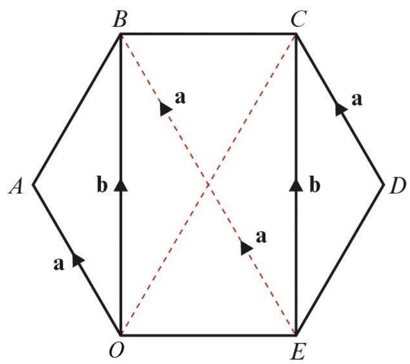
$-\overrightarrow{OP} = 5\mathbf{i} + 6\mathbf{j} - (4\mathbf{i} - 3\mathbf{j})$
$= \mathbf{i} + 9\mathbf{j}$
$\overrightarrow{OP} = -\mathbf{i} - 9\mathbf{j}$
or
$\overrightarrow{OP} = \begin{pmatrix} -1 \\ -9 \end{pmatrix}$

b i $|\overrightarrow{OP}| = \sqrt{(-1)^2 + (-9)^2}$
$= \sqrt{82}$

ii $|\overrightarrow{OQ}| = \sqrt{4^2 + (-3)^2}$
$= \sqrt{25}$
$= 5$

iii $|\overrightarrow{PQ}| = \sqrt{5^2 + 6^2}$
$= \sqrt{61}$

4

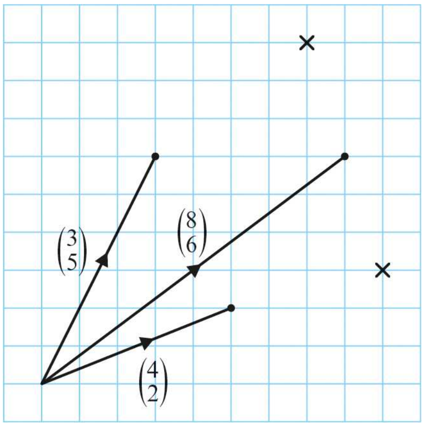


a $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BE} + \overrightarrow{EC}$
$= \mathbf{b} - 2\mathbf{a} + \mathbf{b}$
$= -2\mathbf{a} + 2\mathbf{b}$

b $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$
$= -2\mathbf{a} + 2\mathbf{b} - \mathbf{a}$
$= -3\mathbf{a} + 2\mathbf{b}$

4 c $\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$
$= \mathbf{b} - 2\mathbf{a}$
$= -2\mathbf{a} + \mathbf{b}$

5 The sketch shows the two possible positions of the fourth vertex.



$$\begin{pmatrix}4\\2\end{pmatrix} - \begin{pmatrix}3\\5\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix}$$
$$\begin{pmatrix}8\\6\end{pmatrix} + \begin{pmatrix}1\\-3\end{pmatrix} = \begin{pmatrix}9\\3\end{pmatrix}$$
or
$$-\begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}3\\5\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$$
$$\begin{pmatrix}8\\6\end{pmatrix} + \begin{pmatrix}-1\\3\end{pmatrix} = \begin{pmatrix}7\\9\end{pmatrix}$$
So $\begin{pmatrix}9\\3\end{pmatrix}$ or $\begin{pmatrix}7\\9\end{pmatrix}$

6 a $\overrightarrow{AB} = -(4\mathbf{i} - 5\mathbf{j}) + 6\mathbf{i} + 3\mathbf{j}$
$= 2\mathbf{i} + 8\mathbf{j}$

b $|\overrightarrow{AB}| = \sqrt{2^2 + 8^2}$
$= \sqrt{68}$
$= 2\sqrt{17}$

7 Using the radius of the circle
$|\overrightarrow{OA}| = 3$
Using the position vector
$|\overrightarrow{OA}| = \sqrt{4k^2 + k^2}$
$= \sqrt{5k^2}$
$= \sqrt{5}|k|$
$\sqrt{5}|k| = 3$
$|k| = \dfrac{3}{\sqrt{5}}$
$k = \pm\dfrac{3}{\sqrt{5}}$
Rationalising the denominator
$k = \pm\dfrac{3\sqrt{5}}{5}$

Challenge

Using Pythagoras' Theorem
$x^2 + y^2 = 13$
Solve the equations simultaneously.
Substitute $y = 6 - \frac{3}{2}x$ into $x^2 + y^2 = 13$:
$x^2 + (6 - \frac{3}{2}x)^2 = 13$
$x^2 + 36 - 18x + \frac{9}{4}x^2 - 13 = 0$
$13x^2 - 72x + 92 = 0$
$(13x - 46)(x - 2) = 0$
$x = \frac{46}{13}$ or $x = 2$
When $x = \frac{46}{13}$, $y = \frac{9}{13}$
When $x = 2$, $y = 3$
$\overrightarrow{OB} = \frac{46}{13}\mathbf{i} + \frac{9}{13}\mathbf{j}$ or
$\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{j}$

