Exercise 7B

1 $\mathbf{v}_1$ **i** $8\begin{pmatrix}1\\0\end{pmatrix}+0\begin{pmatrix}0\\1\end{pmatrix}=8\mathbf{i}$

ii $8\begin{pmatrix}1\\0\end{pmatrix}+0\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}8\\0\end{pmatrix}$

$\mathbf{v}_2$ **i** $9\begin{pmatrix}1\\0\end{pmatrix}+3\begin{pmatrix}0\\1\end{pmatrix}=9\mathbf{i}+3\mathbf{j}$

ii $9\begin{pmatrix}1\\0\end{pmatrix}+3\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}9\\3\end{pmatrix}$

$\mathbf{v}_3$ **i** $-4\begin{pmatrix}1\\0\end{pmatrix}+2\begin{pmatrix}0\\1\end{pmatrix}=-4\mathbf{i}+2\mathbf{j}$

ii $-4\begin{pmatrix}1\\0\end{pmatrix}+2\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-4\\2\end{pmatrix}$

$\mathbf{v}_4$ **i** $3\begin{pmatrix}1\\0\end{pmatrix}+5\begin{pmatrix}0\\1\end{pmatrix}=3\mathbf{i}+5\mathbf{j}$

ii $3\begin{pmatrix}1\\0\end{pmatrix}+5\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}3\\5\end{pmatrix}$

$\mathbf{v}_5$ **i** $-3\begin{pmatrix}1\\0\end{pmatrix}-2\begin{pmatrix}0\\1\end{pmatrix}=-3\mathbf{i}-2\mathbf{j}$

ii $-3\begin{pmatrix}1\\0\end{pmatrix}-2\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-3\\-2\end{pmatrix}$

$\mathbf{v}_6$ **i** $0\begin{pmatrix}1\\0\end{pmatrix}-5\begin{pmatrix}0\\1\end{pmatrix}=-5\mathbf{j}$

ii $0\begin{pmatrix}1\\0\end{pmatrix}-5\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\-5\end{pmatrix}$

2 **a** $4\mathbf{a} = 4(2\mathbf{i} + 3\mathbf{j})$
$= 8\mathbf{i} + 12\mathbf{j}$

b $\frac{1}{2}\mathbf{a} = \frac{1}{2}(2\mathbf{i} + 3\mathbf{j})$
$= \mathbf{i} + \frac{3}{2}\mathbf{j}$

2 **c** $-\mathbf{b} = -(4\mathbf{i} - \mathbf{j})$
$= -4\mathbf{i} + \mathbf{j}$

d $2\mathbf{b} + \mathbf{a} = 2(4\mathbf{i} - \mathbf{j}) + (2\mathbf{i} + 3\mathbf{j})$
$= (8\mathbf{i} - 2\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j})$
$= (8 + 2)\mathbf{i} + (-2 + 3)\mathbf{j}$
$= 10\mathbf{i} + \mathbf{j}$

e $3\mathbf{a} - 2\mathbf{b} = 3(2\mathbf{i} + 3\mathbf{j}) - 2(4\mathbf{i} - \mathbf{j})$
$= (6\mathbf{i} + 9\mathbf{j}) - (8\mathbf{i} - 2\mathbf{j})$
$= (6 - 8)\mathbf{i} + (9 + 2)\mathbf{j}$
$= -2\mathbf{i} + 11\mathbf{j}$

f $\mathbf{b} - 3\mathbf{a} = (4\mathbf{i} - \mathbf{j}) - 3(2\mathbf{i} + 3\mathbf{j})$
$= (4\mathbf{i} - \mathbf{j}) - (6\mathbf{i} + 9\mathbf{j})$
$= (4 - 6)\mathbf{i} + (-1 - 9)\mathbf{j}$
$= -2\mathbf{i} - 10\mathbf{j}$

g $4\mathbf{b} - \mathbf{a} = 4(4\mathbf{i} - \mathbf{j}) - (2\mathbf{i} + 3\mathbf{j})$
$= (16\mathbf{i} - 4\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j})$
$= (16 - 2)\mathbf{i} + (-4 - 3)\mathbf{j}$
$= 14\mathbf{i} - 7\mathbf{j}$

h $2\mathbf{a} - 3\mathbf{b} = 2(2\mathbf{i} + 3\mathbf{j}) - 3(4\mathbf{i} - \mathbf{j})$
$= (4\mathbf{i} + 6\mathbf{j}) - (12\mathbf{i} - 3\mathbf{j})$
$= (4 - 12)\mathbf{i} + (6 + 3)\mathbf{j}$
$= -8\mathbf{i} + 9\mathbf{j}$

3 **a** $5\mathrm{a} = 5\begin{pmatrix}9\\7\end{pmatrix}$
$=\begin{pmatrix}45\\35\end{pmatrix}$

b $-\frac{1}{2}\mathbf{c} = -\frac{1}{2}\begin{pmatrix}-8\\-1\end{pmatrix}$
$=\begin{pmatrix}4\\0.5\end{pmatrix}$

c $\mathbf{a} + \mathbf{b} + \mathbf{c} = \begin{pmatrix}9\\7\end{pmatrix}+\begin{pmatrix}11\\-3\end{pmatrix}+\begin{pmatrix}-8\\-1\end{pmatrix}$
$=\begin{pmatrix}12\\3\end{pmatrix}$

3 d $2\mathbf{a} - \mathbf{b} + \mathbf{c} = 2\begin{pmatrix} 9 \\ 7 \end{pmatrix} - \begin{pmatrix} 11 \\ -3 \end{pmatrix} + \begin{pmatrix} -8 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 16 \end{pmatrix}$

e $2\mathbf{b} + 2\mathbf{c} - 3\mathbf{a} = 2\begin{pmatrix} 11 \\ -3 \end{pmatrix} + 2\begin{pmatrix} -8 \\ -1 \end{pmatrix} - 3\begin{pmatrix} 9 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -21 \\ -29 \end{pmatrix}$

f $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathrm{b} = \frac{1}{2}\begin{pmatrix} 9 \\ 7 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 11 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 10 \\ 2 \end{pmatrix}$

4 a $\mathbf{a} + \lambda\mathbf{b} = (2\mathbf{i} + 5\mathbf{j}) + \lambda(3\mathbf{i} - \mathbf{j})$
$= (2 + 3\lambda)\mathbf{i} + (5 - \lambda)\mathbf{j}$
Parallel to $\mathbf{i}$, so $5 - \lambda = 0$, $\lambda = 5$.

b $\mu\mathbf{a} + \mathbf{b} = \mu(2\mathbf{i} + 5\mathbf{j}) + (3\mathbf{i} - \mathbf{j})$
$= (2\mu + 3)\mathbf{i} + (5\mu - 1)\mathbf{j}$
Parallel to $\mathbf{j}$, so $2\mu + 3 = 0$, $\mu = -\frac{3}{2}$

5 a $\mathbf{c} + \lambda\mathbf{d} = (3\mathbf{i} + 4\mathbf{j}) + \lambda(\mathbf{i} - 2\mathbf{j})$
$= (3 + \lambda)\mathbf{i} + (4 - 2\lambda)\mathbf{j}$
Parallel to $\mathbf{i} + \mathbf{j}$, so $3 + \lambda = 4 - 2\lambda$
$3\lambda = 1$, $\lambda = \frac{3}{2}$

b $\mu\mathbf{c} + \mathbf{d} = \mu(3\mathbf{i} + 4\mathbf{j}) + (\mathbf{i} - 2\mathbf{j})$
$= (3\mu + 1)\mathbf{i} + (4\mu - 2)\mathbf{j}$
Parallel to $\mathbf{i} + 3\mathbf{j}$, so $4\mu - 2 = 3(3\mu + 1)$
$4\mu - 2 = 9\mu + 3$
$5\mu = -5$, $\mu = -1$

c $\mathbf{c} - s\mathbf{d} = (3\mathbf{i} + 4\mathbf{j}) - s(\mathbf{i} - 2\mathbf{j})$
$= (3 - s)\mathbf{i} + (4 + 2s)\mathbf{j}$
Parallel to $2\mathbf{i} + \mathbf{j}$, so
$3 - s = 2(4 + 2s)$
$3 - s = 8 + 4\mathbf{i}$
$-5 = 5s$, $s = -1$

d $\mathbf{d} - t\mathbf{c} = (\mathbf{i} - 2\mathbf{j}) - t(3\mathbf{i} + 4\mathbf{j})$
$= (1 - 3t)\,\mathbf{i} + (-2 - 4t)\mathbf{j}$
Parallel to $-2\mathbf{i} + 3\mathbf{j}$, so
$-2(-2 - 4t) = 3(1 - 3t)$
$4 + 8t = 3 - 9t$
$1 = -17t$, $t = -\frac{1}{17}$

6 $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$
$= -(4\mathbf{i} + 3\mathbf{j}) + 5\mathbf{i} + 2\mathbf{j}$
$= \mathbf{i} - \mathbf{j}$

7 a $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$
$= -(2\mathbf{i} + 4\mathbf{j}) + 7\mathbf{i}$
$= 5\mathbf{i} - 4\mathbf{j}$
$= 5\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 4\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$= \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

b $\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AC}$
$= \frac{3}{5}(5\mathbf{i} - 4\mathbf{j})$
$= 3\mathbf{i} - \frac{12}{5}\mathbf{j}$
$= 3\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{12}{5}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$= \begin{pmatrix} 3 \\ -\frac{12}{5} \end{pmatrix}$

c $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$
$= 2\mathbf{i} + 4\mathbf{j} + \frac{3}{5}(5\mathbf{i} - 4\mathbf{j})$
$= 5\mathbf{i} + \frac{8}{5}\mathbf{j}$
$= 5\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{8}{5}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$= \begin{pmatrix} 5 \\ \frac{8}{5} \end{pmatrix}$

8 $\begin{pmatrix} 10 \\ k \end{pmatrix} - 2\begin{pmatrix} j \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$
So $10 - 2j = 2$ and $k - 6 = 5$
$j = 4$ and $k = 11$

9 $\begin{pmatrix} p \\ -q \end{pmatrix} + 2\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$
So $p + 2q = 7$ **(1)**
and $-q + 2p = 4$ **(2)**
Solve the two equations simultaneously.
Multiply equation **(2)** by 2 to give
$-2q + 4p = 8$ **(3)**
Add equations **(1)** and **(3)**
$5p = 15$
$p = 3$ and $q = 2$

10 a The resultant vector

$\mathbf{a} + \mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + p\mathbf{i} - 2p\mathbf{j}$

$= (3 + p)\mathbf{i} - (2 + 2p)\mathbf{j}$

$(3 + p)\mathbf{i} - (2 + 2p)\mathbf{j}$ is parallel to $2\mathbf{i} - 3\mathbf{j}$

so $(3 + p)\mathbf{i} - (2 + 2p)\mathbf{j} = \lambda(2\mathbf{i} - 3\mathbf{j})$

so $3 + p = 2\lambda$ **(1)**

and $2 + 2p = 3\lambda$ **(2)**

Solve the two equations simultaneously.

Multiply equation **(1)** by 2 to give

$6 + 2p = 4\lambda$

$\lambda = 4$

$p = 5$

b When $p = 5$

$(3 + p)\mathbf{i} - (2 + 2p)\mathbf{j} = 8\mathbf{i} - 12\mathbf{j}$