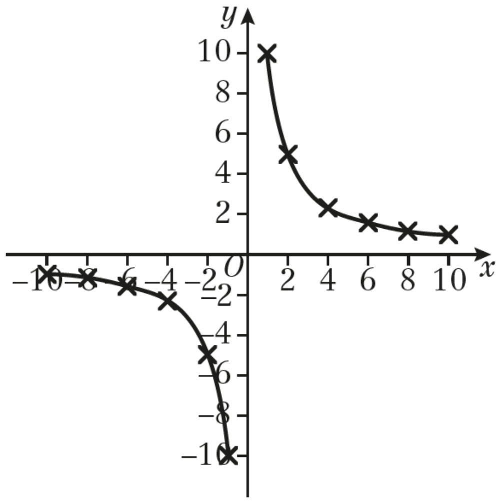
Exercise 3C

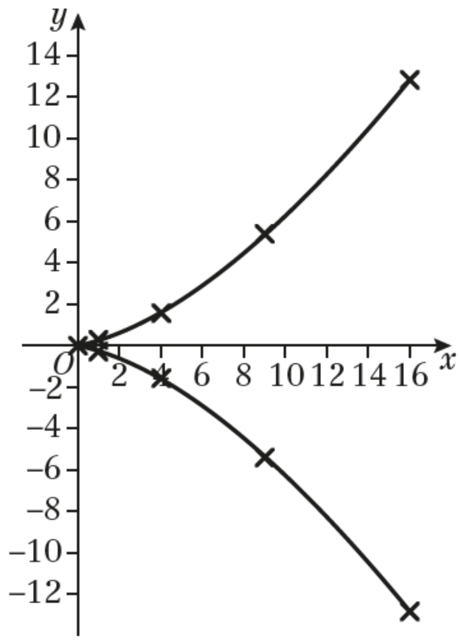
1

t	−5	−4	−3	−2	−1	−0.5	0.5	1	2	3	4	5
$x = 2t$	−10	−8	−6	−4	−2	−1	1	2	4	6	8	10
$y = \frac{5}{t}$	−1	−1.25	−1.67	−2.5	−5	−10	10	5	2.5	1.67	1.25	1



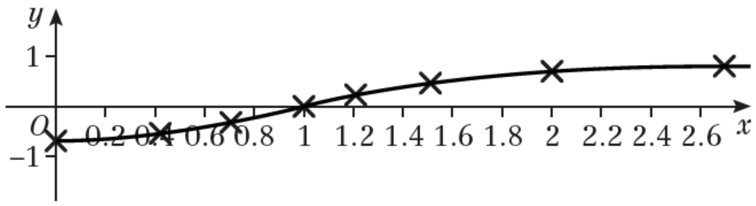
2

t	−4	−3	−2	−1	0	1	2	3	4
$x = t^2$	16	9	4	1	0	1	4	9	16
$y = \frac{t^3}{5}$	−12.8	−5.4	−1.6	−0.2	0	0.2	1.6	5.4	12.8



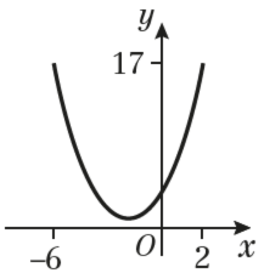
3

t	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$x = \tan t + 1$	0	0.423	0.732	1	1.268	1.577	2	2.732
$y = \sin t$	−0.707	−0.5	−0.259	0	0.259	0.5	0.707	0.866



4 a

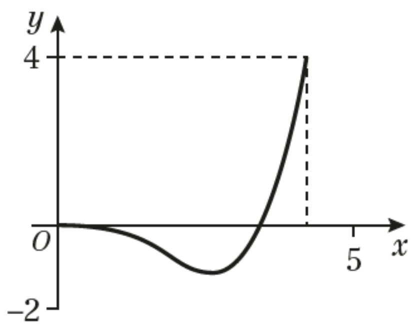
t	−4	−3	−2	−1	0	1	2	3	4
$x = t - 2$	−6	−5	−4	−3	−2	−1	0	1	2
$y = t^2 + 1$	17	10	5	2	1	2	5	10	17



Note that the curve is a parabola with minimum point having coordinates (−2, 1).

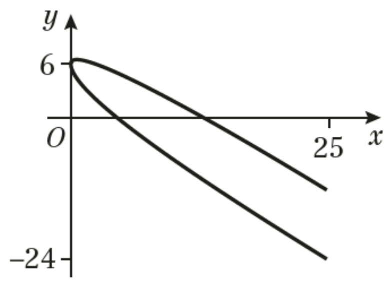
b

t	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$x = 3\sqrt{t}$	0	1.50	2.12	2.60	3.00	3.35	3.67	3.97	4.24
$y = t^3 - 2t$	0	−0.48	−0.88	−1.08	−1.00	−0.55	0.38	1.86	4.00



4 c

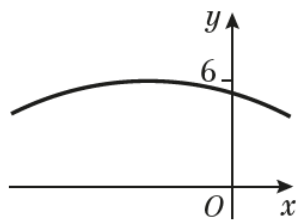
t	−5	−4	−3	−2	−1	0	1	2	3	4	5
$x=t^2$	25	16	9	4	1	0	1	4	9	16	25
$y=(2-t)(t+3)$	−14	−6	0	4	6	6	4	0	−6	−14	−24



Note that the curve crosses the x-axis at $x = 4$ and $x = 9$ and touches the y-axis at $y = 6$.

d

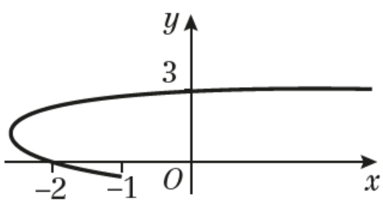
t	$-\frac{\pi}{4}$	$-\frac{3\pi}{16}$	$-\frac{\pi}{8}$	$-\frac{\pi}{16}$	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
$x=2\sin t-1$	−2.41	−2.11	−1.77	−1.39	−1.00	−0.61	−0.23	0.11	0.41
$y=5\cos t+1$	4.54	5.16	5.62	5.90	6.00	5.90	5.62	5.16	4.54



Note the symmetry in the curve about the line $x = -1$ with maximum value $y = 6$.

e

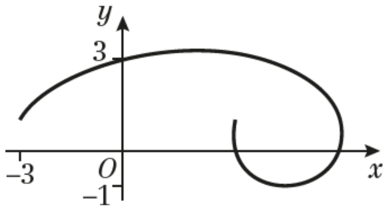
t	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
$x=\sec^2 t-3$	−1.00	−1.67	−1.93	−2.00	−1.93	−1.67	−1.00	1.00	11.93	∞
$y=2\sin t+1$	0.41	0	0.48	1.00	1.52	2.00	2.41	2.73	2.93	3.00



Note that as $y \to 3$ (y approaches 3), $x \to \infty$ (x tends to infinity, that is, gets very large without bound). The line $y = 3$ is an asymptote of the curve.

4 f

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$x = t - 3\cos t$	−3.00	−1.34	1.57	4.48	6.14	6.05	4.71	3.38	3.28
$y = 1 + 2\sin t$	1.00	2.41	3.00	2.41	1.00	−0.41	−1.00	−0.41	1.00



5 a $x = 3 - t \Rightarrow t = 3 - x$ (1)

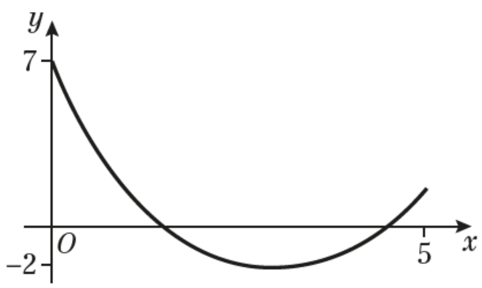
Substitute (1) into
$y = t^2 - 2$

$y = (3 - x)^2 - 2$
or $y = x^2 - 6x + 7$

b

t	−2	−1	0	1	2	3
$x = 3 - t$	5	4	3	2	1	0
$y = t^2 - 2$	2	−1	−2	−1	2	7

The curve is quadratic with a minimum value of $y = -2$ that occurs when $x = 3$.



6 a $x = 9\cos t - 2 \Rightarrow \dfrac{x+2}{9} = \cos t$ (1)

$y = 9\sin t + 1 \Rightarrow \dfrac{y-1}{9} = \sin t$ (2)

Substitute (1) and (2) into
$\cos^2 t + \sin^2 t = 1$

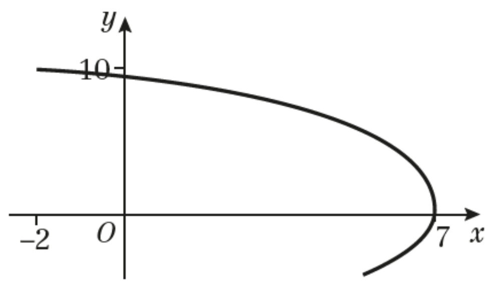
$\left(\dfrac{x+2}{9}\right)^2 + \left(\dfrac{y-1}{9}\right)^2 = 1$

$(x+2)^2 + (y-1)^2 = 81$

So $a = 2$, $b = -1$ and $c = 81$.
The curve is part of a circle, centre $(-2,1)$ and with radius 9 units.

b

t	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
$x = 9\cos t - 2$	5.79	6.69	7.00	6.69	5.79	4.36	2.50	0.33	−2.00
$y = 9\sin t + 1$	−3.50	−1.33	1.00	3.33	5.50	7.36	8.79	9.69	10.00



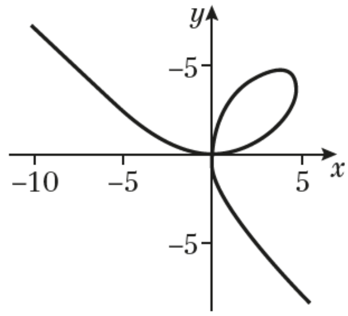
c $r = 9$

$\theta = \dfrac{\pi}{6} + \dfrac{\pi}{2} = \dfrac{2\pi}{3}$

Arc length $= r\theta = 9 \times \dfrac{2\pi}{3} = 6\pi$

Challenge

t	−4	−3	−2	−1.2	−0.8	0	1	2	3	4
$x = \frac{9t}{1+t^3}$	0.57	1.04	2.57	14.84	−14.75	0	4.50	2	0.96	0.55
$y = \frac{9t^2}{1+t^3}$	−2.29	−3.12	−5.14	−17.80	11.80	0	4.50	4	2.89	2.22



As t increases from $t = 0$, the function f(x) creates, in an anticlockwise direction, a small loop in the first quadrant.

To consider the behaviour of the function for $t < 0$, note that the parametric equations are not defined at $t = -1$. It is, therefore, important to investigate the behaviour of the curve as t approaches this value from above and from below. Choose some values of t that are close to −1 and on either side of it. Then observe the behaviour of the curve as t moves away from $t = -1$ in both directions. In particular, check what happens as $t \to -\infty$.

As t approaches −1 from the positive direction the curve heads off to infinity in the second quadrant, and as it approaches −1 from the negative direction it heads off to infinity in the fourth quadrant.

Note that any set of t values can be selected and that they do not need to be spaced equidistantly. In sketching more complicated curves, it is often important to consider additional values of t. Select positions such as $t = -1$ or regions, such as $0 \leqslant t \leqslant 4$, where you notice that the curve is not moving in the same direction.