Exercise 2B

1 $\dfrac{3x^2+x+1}{x^2(x+1)} \equiv \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+1}$

$$\equiv \frac{Ax(x+1)+B(x+1)+Cx^2}{x^2(x+1)}$$

$$3x^2+x+1 \equiv Ax(x+1)+B(x+1)+Cx^2$$

Let $x = 0$:

$0 + 0 + 1 = 0 + B \times 1 + 0$

$B = 1$

Let $x = -1$:

$3 - 1 + 1 = 0 + 0 + C \times (-1)^2$

$C = 3$

Equating terms in x^2:

$3 = A + C$

$3 = A + 3$

$A = 0$

$A = 0, B = 1, C = 3$

2 $\dfrac{-x^2-10x-5}{(x+1)^2(x-1)} \equiv \dfrac{D}{x+1}+\dfrac{E}{(x+1)^2}+\dfrac{F}{x-1}$

$$\equiv \frac{D(x+1)(x-1)+E(x-1)+F(x+1)^2}{(x+1)^2(x-1)}$$

$$-x^2-10x-5 \equiv D(x+1)(x-1)+E(x-1)+F(x+1)^2$$

Let $x = -1$:

$-1 + 10 - 5 = 0 + E \times (-2) + 0$

$4 = -2E$

$E = -2$

Let $x = 1$:

$-1 - 10 - 5 = 0 + 0 + F \times 2^2$

$-16 = 4F$

$F = -4$

Equating terms in x^2:

$-1 = D + F$

$-1 = D - 4$

$D = 3$

$D = 3, E = -2, F = -4$

3 $\dfrac{2x^2+2x-18}{x(x-3)^2} \equiv \dfrac{P}{x}+\dfrac{Q}{x-3}+\dfrac{R}{(x-3)^2}$

$\equiv \dfrac{P(x-3)^2+Qx(x-3)+Rx}{x(x-3)^2}$

$2x^2+2x-18 \equiv P(x-3)^2+Qx(x-3)+Rx$

Let $x = 0$:

$-18 = P \times (-3)^2 + 0 + 0$

$-18 = 9P$

$P = -2$

Let $x = 3$:

$18 + 6 - 18 = 0 + 0 + R \times 3$

$6 = 3R$

$R = 2$

Equating terms in x^2:

$2 = P + Q$

$2 = -2 + Q$

$Q = 4$

$P = -2,\ Q = 4,\ R = 2$

4 First factorise the denominator:

$\dfrac{5x^2-2x-1}{x^3-x^2} \equiv \dfrac{5x^2-2x-1}{x^2(x-1)}$

Then $\dfrac{5x^2-2x-1}{x^2(x-1)} \equiv \dfrac{C}{x}+\dfrac{D}{x^2}+\dfrac{E}{x-1}$

$\equiv \dfrac{Cx(x-1)+D(x-1)+Ex^2}{x^2(x-1)}$

$5x^2-2x-1 \equiv Cx(x-1)+D(x-1)+Ex^2$

Let $x = 0$:

$-1 = 0 + D \times (-1) + 0$

$-1 = -D$

$D = 1$

Let $x = 1$:

$5 - 2 - 1 = 0 + 0 + E \times 1^2$

$E = 2$

Equating terms in x^2:

$5 = C + E$

$5 = C + 2$

$C = 3$

$C = 3,\ D = 1,\ E = 2$

5 $\dfrac{2x}{(x+2)^2} \equiv \dfrac{A}{x+2} + \dfrac{B}{(x+2)^2}$

$\equiv \dfrac{A(x+2)+B}{(x+2)^2}$

$2x \equiv A(x+2)+B$

Let $x = -2$:

$-4 = 0 + B$

$B = -4$

Let $x = 0$:

$0 = 2A + B$

$0 = 2A - 4$

$A = 2$

$A = 2, B = -4$

6 $\dfrac{10x^2-10x+17}{(2x+1)(x-3)^2} \equiv \dfrac{A}{2x+1} + \dfrac{B}{x-3} + \dfrac{C}{(x-3)^2}$

$\equiv \dfrac{A(x-3)^2 + B(2x+1)(x-3) + C(2x+1)}{(2x+1)(x-3)^2}$

$10x^2 - 10x + 17 \equiv A(x-3)^2 + B(2x+1)(x-3) + C(2x+1)$

Let $x = -\frac{1}{2}$:

$\frac{10}{4} + 5 + 17 = A \times \left(-\frac{7}{2}\right)^2 + 0 + 0$

$\frac{98}{4} = \frac{49}{4}A$

$A = 2$

Let $x = 3$:

$90 - 30 + 17 = 0 + 0 + C \times 7$

$77 = 7C$

$C = 11$

Equating terms in x^2:

$10 = A + 2B$

$10 = 2 + 2B$

$B = 4$

$A = 2, B = 4, C = 11$

7 $\dfrac{39x^2+2x+59}{(x+5)(3x-1)^2} \equiv \dfrac{A}{x+5}+\dfrac{B}{3x-1}+\dfrac{C}{(3x-1)^2}$

$\equiv \dfrac{A(3x-1)^2+B(x+5)(3x-1)+C(x+5)}{(x+5)(3x-1)^2}$

$39x^2 + 2x + 59 \equiv A(3x-1)^2 + B(x+5)(3x-1) + C(x+5)$

Let $x = \frac{1}{3}$:

$\frac{39}{9} + \frac{2}{3} + 59 = 0 + 0 + C \times \frac{16}{3}$

$64 = \frac{16}{3}C$

$C = 12$

Let $x = -5$:

$975 - 10 + 59 = A \times (-16)^2 + 0 + 0$

$1024 = 256A$

$A = 4$

Equating terms in x^2:

$39 = 9A + 3B$

$39 = 36 + 3B$

$B = 1$

$A = 4, B = 1, C = 12$

8 a $\dfrac{4x+1}{(x+5)^2} \equiv \dfrac{A}{x+5}+\dfrac{B}{(x+5)^2}$

$\equiv \dfrac{A(x+5)+B}{(x+5)^2}$

$4x + 1 \equiv A(x + 5) + B$

Let $x = -5$:

$-20 + 1 = 0 + B$

$B = -19$

Let $x = 0$:

$1 = 5A + B$

$1 = 5A - 19$

$A = 4$

$\dfrac{4x+1}{(x+5)^2} \equiv \dfrac{4}{x+5}-\dfrac{19}{(x+5)^2}$

8 b $\dfrac{6x^2-x+2}{x(2x-1)^2} \equiv \dfrac{A}{x}+\dfrac{B}{2x-1}+\dfrac{C}{(2x-1)^2}$

$$\equiv \frac{A(2x-1)^2+Bx(2x-1)+Cx}{x(2x-1)^2}$$

$$6x^2-x+2 \equiv A(2x-1)^2+Bx(2x-1)+Cx$$

Let $x = 0$:

$2 = A \times (-1)^2 + 0 + 0$

$A = 2$

Let $x = \frac{1}{2}$:

$\frac{3}{2}-\frac{1}{2}+2 = 0+0+C\times\frac{1}{2}$

$C = 6$

Equating terms in x^2:

$6 = 4A + 2B$

$6 = 8 + 2B$

$B = -1$

So $\dfrac{6x^2-x+2}{x(2x-1)^2} \equiv \dfrac{2}{x}-\dfrac{1}{2x-1}+\dfrac{6}{(2x-1)^2}$