Chapter review 1

1 **a** One or zero of the above statements are false.
b There exist people in cold countries who are happy.
c Either more than or less than a quarter of the people who entered the competition won a prize.

2 **Assumption:** If ab is rational then either a or b is irrational.

Let $a = \frac{c}{d}$ where c and d are integers.

Let $b = \sqrt{k}$, where k is not a square number.

Then $ab = \frac{c\sqrt{k}}{d}$ which is clearly irrational.

This contradicts the original assumption.
Therefore the original statement must be true: if ab is rational then no single number a or b can be irrational.

3 **A** At least one multiple of five is even.

4 **Assumption:** If $a - 2b$ is irrational then neither a nor b is irrational.

Let $a = \frac{c}{d}$ where c and d are integers.

Let $b = \frac{e}{f}$ where e and f are integers.

Then $a - 2b = \frac{c}{d} - \frac{2e}{f}$

$= \frac{cf - 2de}{df}$

which is clearly rational.
This contradicts the original assumption that $a - 2b$ is irrational. Therefore the original statement must be true: if $a - 2b$ is irrational then at least one of a and b is irrational.

5 **Assumption:** There exists integers x and y such that $3x + 18y = 1$

Since 3 is a common factor, $x + 6y = \frac{1}{3}$

Then x and $6y$ are both integers.
The sum $x + 6y$ must also be an integer.

This contradicts the statement that $x + 6y = \frac{1}{3}$

Therefore there exists no integer for which $3x + 18y = 1$

6 **Assumption:** If n^4 is odd then n can be even.
Let $n = 2k$, where k is an integer.
Then $n^4 = (2k)^4 = 16k^4 = 2(8k^4)$ which must be even.
This contradicts the assumption.
Therefore the original statement must be true:
if n^4 is odd then n must be odd.