Core Mathematics 4 Paper L

1. Express

$$\frac{5x}{(x-4)(x+1)} + \frac{3}{(x-2)(x+1)}$$

as a single fraction in its simplest form. [4]

2. A curve has the equation

$$x^2 + 2xy^2 + y = 4.$$

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

3. Evaluate

$$\int_0^{\frac{\pi}{3}} \sin 2x \cos x \;\mathrm{d}x.$$ [5]

4. A curve has parametric equations

$$x = \cos 2t, \quad y = \operatorname{cosec} t, \quad 0 < t < \frac{\pi}{2}.$$

The point P on the curve has x-coordinate $\frac{1}{2}$.

(i) Find the value of the parameter t at P. [2]

(ii) Show that the tangent to the curve at P has the equation

$$y = 2x + 1.$$ [5]

5. *(i)* Express $\frac{2+20x}{1+2x-8x^2}$ as a sum of partial fractions. [3]

(ii) Hence find the series expansion of $\frac{2+20x}{1+2x-8x^2}$, $|x| < \frac{1}{4}$, in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [5]

6. Use the substitution $x = 2\tan u$ to show that

$$\int_0^2 \frac{x^2}{x^2+4}\;\mathrm{d}x = \tfrac{1}{2}(4-\pi).$$ [8]

7. A straight road passes through villages at the points A and B with position vectors $(9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k})$ and $(4\mathbf{j} + \mathbf{k})$ respectively, relative to a fixed origin.

The road ends at a junction at the point C with another straight road which lies along the line with equation

$$\mathbf{r} = (2\mathbf{i} + 16\mathbf{j} - \mathbf{k}) + t(-5\mathbf{i} + 3\mathbf{j}),$$

where t is a scalar parameter.

(i) Find the position vector of C. [5]

Given that 1 unit on each coordinate axis represents 200 metres,

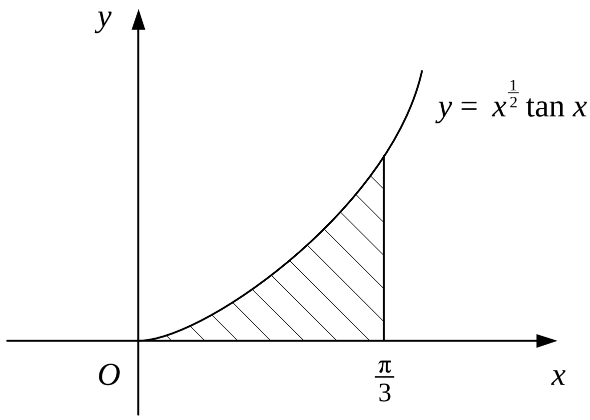
(ii) find the distance, in kilometres, from the village at A to the junction at C. [4]

8. *(i)* Find $\int \tan^2 x \; \mathrm{d}x$. [3]

(ii) Show that

$$\int \tan x \; \mathrm{d}x = \ln |\sec x| + c,$$

where c is an arbitrary constant. [4]



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} \tan x$.

The shaded region bounded by the curve, the x-axis and the line $x = \frac{\pi}{3}$ is rotated through 360° about the x-axis.

(iii) Show that the volume of the solid formed is $\frac{1}{18}\pi^2(6\sqrt{3} - \pi) - \pi \ln 2$. [5]

Turn over

9. An entomologist is studying the population of insects in a colony.

Initially there are 300 insects in the colony and in a model, the entomologist assumes that the population, P, at time t weeks satisfies the differential equation

$$\frac{dP}{dt} = kP,$$

where k is a constant.

(i) Find an expression for P in terms of k and t. [5]

Given that after one week there are 360 insects in the colony,

(ii) find the value of k to 3 significant figures. [2]

Given also that after two and three weeks there are 440 and 600 insects respectively,

(iii) comment on suitability of the modelling assumption. [2]

An alternative model assumes that

$$\frac{dP}{dt} = P(0.4 - 0.25 \cos 0.5t).$$

(iv) Using the initial data, $P = 300$ when $t = 0$, solve this differential equation. [3]

(v) Compare the suitability of the two models. [2]