C4 Paper L – Marking Guide

1.	$= \frac{5x(x-2)+3(x-4)}{(x-4)(x+1)(x-2)} = \frac{5x^2-7x-12}{(x-4)(x+1)(x-2)}$	M1 A1
	$= \frac{(5x-12)(x+1)}{(x-4)(x+1)(x-2)} = \frac{5x-12}{(x-4)(x-2)}$	M1 A1 **(4)**
2.	$2x + 2y^2 + 2x \times 2y\frac{dy}{dx} + \frac{dy}{dx} = 0$	M1 A2
	$\frac{dy}{dx} = -\frac{2x+2y^2}{4xy+1}$	M1 A1 **(5)**
3.	$= \int_0^{\frac{\pi}{3}} 2\sin x\cos^2 x \; dx$	M1
	$= [-\frac{2}{3}\cos^3 x]_0^{\frac{\pi}{3}}$	M1 A1
	$= -\frac{1}{12} - (-\frac{2}{3}) = \frac{7}{12}$	M1 A1 **(5)**
4.	*(i)* $\cos 2t = \frac{1}{2}$, $2t = \frac{\pi}{3}$, $t = \frac{\pi}{6}$	M1 A1
	(ii) $\frac{dx}{dt} = -2\sin 2t$, $\frac{dy}{dt} = -\text{cosec}\, t\cot t$	M1
	$\frac{dy}{dx} = \frac{-\text{cosec}\, t\cot t}{-2\sin 2t}$	M1 A1
	$t = \frac{\pi}{6}$, $y = 2$, grad $= 2$	
	$\therefore\; y - 2 = 2(x - \frac{1}{2})$	M1
	$y = 2x + 1$	A1 **(7)**
5.	*(i)* $\frac{2+20x}{1+2x-8x^2} = \frac{2+20x}{(1-2x)(1+4x)} \equiv \frac{A}{1-2x} + \frac{B}{1+4x}$	
	$2 + 20x \equiv A(1+4x) + B(1-2x)$	M1
	$x = \frac{1}{2} \;\Rightarrow\; 12 = 3A \;\Rightarrow\; A = 4$	A1
	$x = -\frac{1}{4} \;\Rightarrow\; -3 = \frac{3}{2}B \;\Rightarrow\; B = -2$	A1
	$\frac{2+20x}{1+2x-8x^2} \equiv \frac{4}{1-2x} - \frac{2}{1+4x}$	
	(ii) $\frac{2+20x}{1+2x-8x^2} = 4(1-2x)^{-1} - 2(1+4x)^{-1}$	
	$(1-2x)^{-1} = 1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-2x)^3 + \ldots$	M1
	$= 1 + 2x + 4x^2 + 8x^3 + \ldots$	A1
	$(1+4x)^{-1} = 1 + (-1)(4x) + \frac{(-1)(-2)}{2}(4x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(4x)^3 + \ldots$	
	$= 1 - 4x + 16x^2 - 64x^3 + \ldots$	A1
	$\frac{2+20x}{1+2x-8x^2} = 4(1 + 2x + 4x^2 + 8x^3 + \ldots) - 2(1 - 4x + 16x^2 - 64x^3 + \ldots)$	M1
	$= 2 + 16x - 16x^2 + 160x^3 + \ldots$	A1 **(8)**
6.	$x = 2\tan u \;\Rightarrow\; \frac{dx}{du} = 2\sec^2 u$, $\quad x = 0 \Rightarrow u = 0$, $x = 2 \Rightarrow u = \frac{\pi}{4}$	M1 B1
	$I = \int_0^{\frac{\pi}{4}} \frac{4\tan^2 u}{4\sec^2 u} \times 2\sec^2 u \; du = \int_0^{\frac{\pi}{4}} 2\tan^2 u \; du$	M1 A1
	$= \int_0^{\frac{\pi}{4}} (2\sec^2 u - 2) \; du$	M1
	$= [2\tan u - 2u]_0^{\frac{\pi}{4}}$	M1
	$= (2 - \frac{\pi}{2}) - (0) = \frac{1}{2}(4 - \pi)$	M1 A1 **(8)**

7.	*(i)*	$\overrightarrow{AB} = (4\mathbf{j} + \mathbf{k}) - (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) = (-9\mathbf{i} + 12\mathbf{j} - \mathbf{k})$	M1	
		$\therefore\ \mathbf{r} = (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) + s(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k})$	A1	
		at C, $2 - s = -1$, $s = 3$	M1 A1	
		$\therefore\ \overrightarrow{OC} = (9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) + 3(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k}) = (-18\mathbf{i} + 28\mathbf{j} - \mathbf{k})$	A1	
	(ii)	$\overrightarrow{AC} = 3(-9\mathbf{i} + 12\mathbf{j} - \mathbf{k})$, $AC = 3\sqrt{81+144+1} = 45.10$	M1 A1	
		$\therefore$ distance $= 200 \times 45.10 = 9020$ m $= 9.02$ km (3sf)	M1 A1	**(9)**
8.	*(i)*	$= \int (\sec^2 x - 1)\ dx$	M1	
		$= \tan x - x + c$	M1 A1	
	(ii)	$= \int \frac{\sin x}{\cos x}\ dx$, let $u = \cos x$, $\frac{du}{dx} = -\sin x$	M1	
		$= \int \frac{1}{u} \times (-1)\ du = -\int \frac{1}{u}\ du$	A1	
		$= -\ln\lvert u\rvert + c = \ln\lvert u^{-1}\rvert + c = \ln\lvert \sec x\rvert + c$	M1 A1	
	(iii)	volume $= \pi\int_0^{\frac{\pi}{3}} x\tan^2 x\ dx$		
		$u = x$, $u' = 1$, $v' = \tan^2 x$, $v = \tan x - x$	M1	
		$I = x(\tan x - x) - \int (\tan x - x)\ dx$	A1	
		$= x\tan x - x^2 - \ln\lvert \sec x\rvert + \frac{1}{2}x^2 + c$	A1	
		volume $= \pi[x\tan x - \frac{1}{2}x^2 - \ln\lvert \sec x\rvert]_0^{\frac{\pi}{3}}$		
		$= \pi\{(\frac{1}{3}\sqrt{3}\,\pi - \frac{1}{18}\pi^2 - \ln 2) - (0)\}$	M1	
		$= \frac{1}{18}\pi^2(6\sqrt{3} - \pi) - \pi\ln 2$	A1	**(12)**
9.	*(i)*	$\int \frac{1}{P}\ dP = \int k\ dt$	M1	
		$\ln\lvert P\rvert = kt + c$	A1	
		$t = 0$, $P = 300 \Rightarrow \ln 300 = c$	M1	
		$\ln\lvert P\rvert = kt + \ln 300$		
		$\ln\lvert \frac{P}{300}\rvert = kt$, $\frac{P}{300} = e^{kt}$, $P = 300e^{kt}$	M1 A1	
	(ii)	$t = 1$, $P = 360 \Rightarrow 360 = 300e^k$	M1	
		$k = \ln\frac{6}{5} = 0.182$ (3sf)	A1	
	(iii)	$P = 300e^{0.1823t}$		
		when $t = 2$, $P = 432$; when $t = 3$, $P = 518$	B1	
		model does not seem suitable as data diverges from predictions	B1	
	(iv)	$\int \frac{1}{P}\ dP = \int (0.4 - 0.25\cos 0.5t)\ dt$		
		$\ln\lvert P\rvert = 0.4t - 0.5\sin 0.5t + c$	M1	
		$t = 0$, $P = 300 \Rightarrow \ln 300 = c$	M1	
		$\ln\lvert \frac{P}{300}\rvert = 0.4t - 0.5\sin 0.5t$ $[\,P = 300e^{0.4t - 0.5\sin 0.5t}\,]$	A1	
	(v)	second model: $t = 1, 2, 3 \Rightarrow P = 352, 438, 605$	B1	
		the second model seems more suitable as it fits the data better	B1	**(14)**
			Total	**(72)**