Core Mathematics 4 Paper K

1. Evaluate

$$\int_0^{\frac{\pi}{2}} x \cos x \; dx,$$

giving your answer in terms of π. [5]

2. *(i)* Find the binomial expansion of $(2 - 3x)^{-3}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [5]

(ii) State the set of values of x for which your expansion is valid. [1]

3. *(i)* Express $\dfrac{x+11}{(x+4)(x-3)}$ as a sum of partial fractions. [3]

(ii) Evaluate

$$\int_0^2 \frac{x+11}{(x+4)(x-3)} \; dx,$$

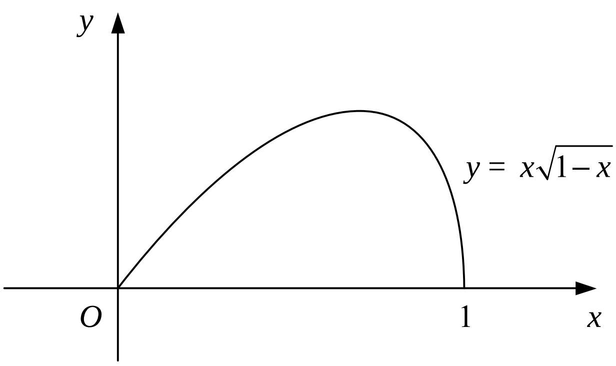
giving your answer in the form $\ln k$, where k is an exact simplified fraction. [4]

4. A curve has the equation

$$4x^2 - 2xy - y^2 + 11 = 0.$$

Find an equation for the normal to the curve at the point with coordinates $(-1, -3)$. [7]

5.



The diagram shows the curve with equation $y = x\sqrt{1-x}$, $0 \le x \le 1$.

Use the substitution $u^2 = 1 - x$ to show that the area of the region bounded by the curve and the x-axis is $\frac{4}{15}$. [7]

6. The number of people, n, in a queue at a Post Office t minutes after it opens is modelled by the differential equation

$$\frac{dn}{dt} = e^{0.5t} - 5, \quad t \geq 0.$$

(i) Find, to the nearest second, the time when the model predicts that there will be the least number of people in the queue. [3]

(ii) Given that there are 20 people in the queue when the Post Office opens, solve the differential equation. [4]

(iii) Explain why this model would not be appropriate for large values of t. [1]

7. *(i)* Show that $(2x + 3)$ is a factor of $(2x^3 - x^2 + 4x + 15)$ and hence, simplify

$$\frac{2x^2 + x - 3}{2x^3 - x^2 + 4x + 15}.$$ [5]

(ii) Show that

$$\int_2^5 \frac{2x^2 + x - 3}{2x^3 - x^2 + 4x + 15} \, dx = \ln k,$$

where k is an integer. [4]

8. The points A and B have coordinates $(3, 9, -7)$ and $(13, -6, -2)$ respectively.

(i) Find, in vector form, an equation for the line l which passes through A and B. [2]

(ii) Show that the point C with coordinates $(9, 0, -4)$ lies on l. [2]

The point D is the point on l closest to the origin, O.

(iii) Find the coordinates of D. [3]

(iv) Find the area of triangle OAB to 3 significant figures. [3]

Turn over

9. A curve has parametric equations

$$x = \sec\theta + \tan\theta, \quad y = \csc\theta + \cot\theta, \quad 0 < \theta < \frac{\pi}{2}.$$

(i) Show that $x + \frac{1}{x} = 2\sec\theta$. [4]

Given that $y + \frac{1}{y} = 2\csc\theta$,

(ii) find a cartesian equation for the curve. [3]

(iii) Show that $\frac{dx}{d\theta} = \frac{1}{2}(x^2 + 1)$. [3]

(iv) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [3]