C4 Paper K – Marking Guide

1.		$u = x,\ \ u' = 1,\ \ v' = \cos x,\ \ v = \sin x$	M1	
		$I = [x \sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \ \, dx$	A1	
		$= [x \sin x + \cos x]_0^{\frac{\pi}{2}}$	A1	
		$= (\frac{\pi}{2} + 0) - (0 + 1) = \frac{\pi}{2} - 1$	M1 A1	**(5)**
2.	*(i)*	$= 2^{-3}(1 - \frac{3}{2}x)^{-3} = \frac{1}{8}(1 - \frac{3}{2}x)^{-3}$	B1	
		$= \frac{1}{8}[1 + (-3)(-\frac{3}{2}x) + \frac{(-3)(-4)}{2}(-\frac{3}{2}x)^2 + \frac{(-3)(-4)(-5)}{3\times 2}(-\frac{3}{2}x)^3 + \ldots]$	M1	
		$= \frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2 + \frac{135}{32}x^3 + \ldots$	A3	
	(ii)	$\lvert x \rvert < \frac{2}{3}$	B1	**(6)**
3.	*(i)*	$\frac{x+11}{(x+4)(x-3)} \equiv \frac{A}{x+4} + \frac{B}{x-3}, \qquad x + 11 \equiv A(x-3) + B(x+4)$	M1	
		$x = -4 \ \Rightarrow \ 7 = -7A \ \Rightarrow \ A = -1$	A1	
		$x = 3 \ \Rightarrow \ 14 = 7B \ \Rightarrow \ B = 2$	A1	
		$\frac{x+11}{(x+4)(x-3)} \equiv \frac{2}{x-3} - \frac{1}{x+4}$		
	(ii)	$= \int_0^2 (\frac{2}{x-3} - \frac{1}{x+4}) \ dx$		
		$= [2 \ln\lvert x-3\rvert - \ln\lvert x+4\rvert]_0^2$	M1 A1	
		$= (0 - \ln 6) - (2 \ln 3 - \ln 4)$	M1	
		$= \ln \frac{2}{27}$	A1	**(7)**
4.		$8x - 2y - 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	M1 A1	
		$(-1, -3) \ \Rightarrow \ -8 + 6 + 2\frac{dy}{dx} + 6\frac{dy}{dx} = 0, \quad \frac{dy}{dx} = \frac{1}{4}$	M1 A1	
		grad of normal $= -4$	M1	
		$\therefore \ \ y + 3 = -4(x + 1) \qquad [\, y = -4x - 7 \,]$	M1 A1	**(7)**
5.		$u^2 = 1 - x \ \Rightarrow \ x = 1 - u^2, \ \ \frac{dx}{du} = -2u$	M1	
		$x = 0 \ \Rightarrow \ u = 1, \ \ x = 1 \ \Rightarrow \ u = 0$	B1	
		area $= \int_0^1 x\sqrt{1-x} \ \ dx = \int_1^0 (1 - u^2) \times u \times (-2u) \ \ du$	M1	
		$= \int_0^1 (2u^2 - 2u^4) \ \ du$	A1	
		$= [\frac{2}{3}u^3 - \frac{2}{5}u^5]_0^1$	M1	
		$= (\frac{2}{3} - \frac{2}{5}) - (0) = \frac{4}{15}$	M1 A1	**(7)**
6.	*(i)*	$\frac{dn}{dt} = 0 \ \Rightarrow \ e^{0.5t} = 5$	M1	
		$t = 2 \ln 5 = 3.219$ mins $= 3$ mins 13 secs	M1 A1	
	(ii)	$\int dn = \int (e^{0.5t} - 5) \ \ dt$		
		$n = 2e^{0.5t} - 5t + c$	M1 A1	
		$t = 0, \ \ n = 20 \ \Rightarrow \ 20 = 2 + c, \quad c = 18$	M1	
		$n = 2e^{0.5t} - 5t + 18$	A1	
	(iii)	as t increases, n rapidly becomes very large $\therefore$ not realistic	B1	**(8)**

7. *(i)* let $\text{f}(x) = 2x^3 - x^2 + 4x + 15$

$\text{f}(-\tfrac{3}{2}) = -\tfrac{27}{4} - \tfrac{9}{4} - 6 + 15 = 0 \quad \therefore (2x + 3)$ is a factor — B1

$$\begin{array}{r|l} & x^2 - 2x + 5 \\ 2x+3 & 2x^3 - x^2 + 4x + 15 \\ & 2x^3 + 3x^2 \\ & \quad -4x^2 + 4x \\ & \quad -4x^2 - 6x \\ & \qquad 10x + 15 \\ & \qquad 10x + 15 \end{array}$$

— M1 A1

$\therefore \text{f}(x) = (2x + 3)(x^2 - 2x + 5)$

$\therefore \dfrac{2x^2+x-3}{2x^3-x^2+4x+15} = \dfrac{(2x+3)(x-1)}{(2x+3)(x^2-2x+5)} = \dfrac{x-1}{x^2-2x+5}$ — M1 A1

(ii) $= \displaystyle\int_2^5 \dfrac{x-1}{x^2-2x+5}\,dx = [\tfrac{1}{2}\ln|x^2 - 2x + 5|]_2^5$ — M1 A1

$= \tfrac{1}{2}(\ln 20 - \ln 5) = \tfrac{1}{2}\ln 4 = \ln 2$ — M1 A1 **(9)**

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8. *(i)* $\overrightarrow{AB} = \begin{pmatrix}10\\-15\\5\end{pmatrix}, \quad \therefore \mathbf{r} = \begin{pmatrix}3\\9\\-7\end{pmatrix} + \lambda\begin{pmatrix}2\\-3\\1\end{pmatrix}$ — M1 A1

(ii) $3 + 2\lambda = 9 \quad \therefore \lambda = 3$ — M1

when $\lambda = 3$, $\mathbf{r} = \begin{pmatrix}3\\9\\-7\end{pmatrix} + 3\begin{pmatrix}2\\-3\\1\end{pmatrix} = \begin{pmatrix}9\\0\\-4\end{pmatrix} \quad \therefore (9, 0, -4)$ lies on l — A1

(iii) $\overrightarrow{OD} = \begin{pmatrix}3+2\lambda\\9-3\lambda\\-7+\lambda\end{pmatrix} \quad \therefore \begin{pmatrix}3+2\lambda\\9-3\lambda\\-7+\lambda\end{pmatrix}\cdot\begin{pmatrix}2\\-3\\1\end{pmatrix} = 0$ — M1

$6 + 4\lambda - 27 + 9\lambda - 7 + \lambda = 0$

$\lambda = 2 \quad \therefore \overrightarrow{OD} = \begin{pmatrix}7\\3\\-5\end{pmatrix}$, $D\,(7, 3, -5)$ — M1 A1

(iv) $AB = \sqrt{100+225+25} = \sqrt{350}$, $OD = \sqrt{49+9+25} + \sqrt{83}$ — M1

area $= \tfrac{1}{2} \times \sqrt{350} \times \sqrt{83} = 85.2$ (3sf) — M1 A1 **(10)**

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9. *(i)* $x + \dfrac{1}{x} = \sec\theta + \tan\theta + \dfrac{1}{\sec\theta+\tan\theta} = \dfrac{(\sec\theta+\tan\theta)^2+1}{\sec\theta+\tan\theta}$ — M1

$= \dfrac{\sec^2\theta+2\sec\theta\tan\theta+\tan^2\theta+1}{\sec\theta+\tan\theta} = \dfrac{2\sec^2\theta+2\sec\theta\tan\theta}{\sec\theta+\tan\theta}$ — M1

$= \dfrac{2\sec\theta(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta}$ — M1

$= 2\sec\theta$ — A1

(ii) $\dfrac{x^2+1}{x} = \dfrac{2}{\cos\theta} \Rightarrow \cos\theta = \dfrac{2x}{x^2+1}$ — M1

$\dfrac{y^2+1}{y} = \dfrac{2}{\sin\theta} \Rightarrow \sin\theta = \dfrac{2y}{y^2+1}, \quad \therefore \dfrac{4x^2}{(x^2+1)^2} + \dfrac{4y^2}{(y^2+1)^2} = 1$ — M1 A1

(iii) $\dfrac{dx}{d\theta} = \sec\theta\tan\theta + \sec^2\theta$ — M1

$= \sec\theta(\tan\theta + \sec\theta) = \dfrac{x^2+1}{2x} \times x = \tfrac{1}{2}(x^2 + 1)$ — M1 A1

(iv) $\dfrac{dy}{d\theta} = -\text{cosec}\,\theta\cot\theta - \text{cosec}^2\,\theta$ — M1

$= -\text{cosec}\,\theta(\cot\theta + \text{cosec}\,\theta) = -\dfrac{y^2+1}{2y} \times y = -\tfrac{1}{2}(y^2 + 1)$

$\therefore \dfrac{dy}{dx} = -\dfrac{y^2+1}{x^2+1}$ — M1 A1 **(13)**

Total **(72)**