

Core Mathematics 4 Paper J

1. Show that

$$\int_2^4 x(x^2 - 4)^{\frac{1}{2}} dx = 8\sqrt{3}. \quad [5]$$

2. (i) Simplify

$$\frac{2x^2 + 3x - 9}{2x^2 - 7x + 6}. \quad [2]$$

(ii) Find the quotient and remainder when $(2x^4 - 1)$ is divided by $(x^2 - 2)$. [4]

3. A curve has the equation

$$2 \sin 2x - \tan y = 0.$$

(i) Find an expression for $\frac{dy}{dx}$ in its simplest form in terms of x and y . [4]

(ii) Show that the tangent to the curve at the point $(\frac{\pi}{6}, \frac{\pi}{3})$ has the equation

$$y = \frac{1}{2}x + \frac{\pi}{4}. \quad [3]$$

4. The gradient at any point (x, y) on a curve is proportional to \sqrt{y} .

Given that the curve passes through the point with coordinates $(0, 4)$,

(i) show that the equation of the curve can be written in the form

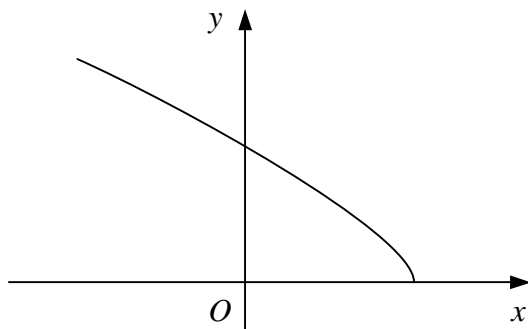
$$2\sqrt{y} = kx + 4,$$

where k is a positive constant. [5]

Given also that the curve passes through the point with coordinates $(2, 9)$,

(ii) find the equation of the curve in the form $y = f(x)$. [3]

5.



The diagram shows the curve with parametric equations

$$x = 2 - t^2, \quad y = t(t + 1), \quad t \geq 0.$$

- (i) Find the coordinates of the points where the curve meets the coordinate axes. [3]
- (ii) Find an equation for the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$. [6]

6.

$$f(x) = \frac{1+3x}{(1-x)(1-3x)}, \quad |x| < \frac{1}{3}.$$

- (i) Find the values of the constants A and B such that

$$f(x) = \frac{A}{1-x} + \frac{B}{1-3x}. \quad [3]$$

- (ii) Evaluate

$$\int_0^{\frac{1}{4}} f(x) \, dx,$$

giving your answer as a single logarithm. [4]

- (iii) Find the series expansion of $f(x)$ in ascending powers of x up to and including the term in x^3 , simplifying each coefficient. [5]

Turn over

7. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$$

and

$$\mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -6 \end{pmatrix} + t \begin{pmatrix} 3 \\ a \\ b \end{pmatrix},$$

where a and b are constants and s and t are scalar parameters.

Given that the two lines are perpendicular,

(i) find a linear relationship between a and b . [2]

Given also that the two lines intersect,

(ii) find the values of a and b , [8]

(iii) find the coordinates of the point where they intersect. [2]

8. (i) Find

$$\int x^2 e^{\frac{1}{2}x} dx. \quad [6]$$

(ii) Using the substitution $u = \sin t$, evaluate

$$\int_0^{\frac{\pi}{2}} \sin^2 2t \cos t dt. \quad [7]$$