C4 Paper J – Marking Guide

1.		$= [\frac{1}{3}(x^2-4)^{\frac{3}{2}}]_2^4$	M1 A1	
		$= \frac{1}{3}(12^{\frac{3}{2}} - 0)$	M1	
		$= \frac{1}{3} \times (2\sqrt{3})^3 = \frac{1}{3} \times 8 \times 3\sqrt{3} = 8\sqrt{3}$	M1 A1	**(5)**
2.	*(i)*	$= \frac{(2x-3)(x+3)}{(2x-3)(x-2)} = \frac{x+3}{x-2}$	M1 A1	
	(ii)	$\begin{array}{r\|rrrrrrrr} & 2x^2 & + & 0x & + & 4 & & & \\ \hline x^2-2 & 2x^4 & +0x^3 & + & 0x^2 & + & 0x & - & 1 \\ & 2x^4 & +0x^3 & - & 4x^2 & & & & \\ \hline & & & & 4x^2 & + & 0x & - & 1 \\ & & & & 4x^2 & + & 0x & - & 8 \\ \hline & & & & & & & & 7 \end{array}$	M2	
		$\therefore$ quotient $= 2x^2 + 4$, remainder $= 7$	A2	**(6)**
3.	*(i)*	$4\cos 2x - \sec^2 y \frac{dy}{dx} = 0$	M1 A1	
		$\frac{dy}{dx} = 4\cos 2x \cos^2 y$	M1 A1	
	(ii)	grad $= 4 \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{2}$	B1	
		$\therefore\ y - \frac{\pi}{3} = \frac{1}{2}(x - \frac{\pi}{6})$	M1	
		$y - \frac{\pi}{3} = \frac{1}{2}x - \frac{\pi}{12}$		
		$y = \frac{1}{2}x + \frac{\pi}{4}$	A1	**(7)**
4.	*(i)*	$\frac{dy}{dx} = k\sqrt{y}$		
		$\int y^{-\frac{1}{2}}\ dy = \int k\ dx$	M1	
		$2y^{\frac{1}{2}} = kx + c$	M1 A1	
		$(0, 4) \Rightarrow 4 = c$	M1	
		$\therefore\ 2\sqrt{y} = kx + 4$	A1	
	(ii)	$(2, 9) \Rightarrow 6 = 2k + 4,\quad k = 1$	M1	
		$\therefore\ 2\sqrt{y} = x + 4,\quad \sqrt{y} = \frac{1}{2}(x+4)$	M1	
		$y = \frac{1}{4}(x+4)^2$	A1	**(8)**
5.	*(i)*	$x = 0 \Rightarrow t^2 = 2$		
		$t \geq 0\ \therefore\ t = \sqrt{2}\quad \therefore (0, 2 + \sqrt{2})$	M1 A1	
		$y = 0 \Rightarrow t(t+1) = 0$		
		$t \geq 0\ \therefore\ t = 0\quad \therefore (2, 0)$	A1	
	(ii)	$\frac{dx}{dt} = -2t,\quad \frac{dy}{dt} = 2t + 1$	M1	
		$\frac{dy}{dx} = -\frac{2t+1}{2t}$	M1 A1	
		$t = 2,\ x = -2,\ y = 6,$ grad $= -\frac{5}{4}$	M1	
		$\therefore\ y - 6 = -\frac{5}{4}(x+2)$	M1	
		$4y - 24 = -5x - 10$		
		$5x + 4y - 14 = 0$	A1	**(9)**

6.	*(i)*	$1 + 3x \equiv A(1 - 3x) + B(1 - x)$	M1	
		$x = 1 \quad \Rightarrow \quad 4 = -2A \quad \Rightarrow \quad A = -2$	A1	
		$x = \frac{1}{3} \quad \Rightarrow \quad 2 = \frac{2}{3}B \quad \Rightarrow \quad B = 3$	A1	
	(ii)	$= \int_0^{\frac{1}{4}} \left(\frac{3}{1-3x} - \frac{2}{1-x}\right)\,dx$		
		$= [-\ln\lvert 1 - 3x\rvert + 2\ln\lvert 1 - x\rvert]_0^{\frac{1}{4}}$	M1 A1	
		$= (-\ln\frac{1}{4} + 2\ln\frac{3}{4}) - (0)$	M1	
		$= \ln\frac{9}{16} - \ln\frac{1}{4} = \ln\frac{9}{4}$	A1	
	(iii)	$f(x) = 3(1 - 3x)^{-1} - 2(1 - x)^{-1}$		
		$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots$	B1	
		$(1 - 3x)^{-1} = 1 + 3x + (3x)^2 + (3x)^3 + \ldots = 1 + 3x + 9x^2 + 27x^3 + \ldots$	M1 A1	
		$\therefore\; f(x) = 3(1 + 3x + 9x^2 + 27x^3 + \ldots) - 2(1 + x + x^2 + x^3 + \ldots)$	M1	
		$= 1 + 7x + 25x^2 + 79x^3 + \ldots$	A1	**(12)**
7.	*(i)*	$\begin{pmatrix}1\\4\\5\end{pmatrix}.\begin{pmatrix}3\\a\\b\end{pmatrix} = 0 \quad \therefore\; 3 + 4a + 5b = 0$	M1 A1	
	(ii)	$4 + s = -3 + 3t \quad (1)$		
		$1 + 4s = 1 + at \quad (2)$		
		$1 + 5s = -6 + bt \quad (3)$	B1	
		$(1) \Rightarrow s = 3t - 7$	M1	
		sub. (2) $\Rightarrow\; 1 + 4(3t - 7) = 1 + at$		
		$12t - 28 = at, \quad t(12 - a) = 28, \quad t = \frac{28}{12-a}$	M1 A1	
		sub. (3) $\Rightarrow\; 1 + 5(3t - 7) = -6 + bt$		
		$15t - 28 = bt, \quad t(15 - b) = 28, \quad t = \frac{28}{15-b}$	A1	
		$\frac{28}{12-a} = \frac{28}{15-b}, \quad 12 - a = 15 - b, \quad b = a + 3$	M1	
		sub *(a)* $\Rightarrow\; 3 + 4a + 5(a + 3) = 0, \;\; a = -2,\; b = 1$	M1 A1	
	(iii)	$t = 2 \;\;\therefore\; \mathbf{r} = \begin{pmatrix}-3\\1\\-6\end{pmatrix} + 2\begin{pmatrix}3\\-2\\1\end{pmatrix} = \begin{pmatrix}3\\-3\\-4\end{pmatrix}, \;\;\therefore\; (3, -3, -4)$	M1 A1	**(12)**
8.	*(i)*	$u = x^2,\; u' = 2x,\; v' = e^{\frac{1}{2}x},\; v = 2e^{\frac{1}{2}x}$	M1	
		$I = 2x^2 e^{\frac{1}{2}x} - \int 4x e^{\frac{1}{2}x}\,dx$	A2	
		$u = 4x,\; u' = 4,\; v' = e^{\frac{1}{2}x},\; v = 2e^{\frac{1}{2}x}$	M1	
		$I = 2x^2 e^{\frac{1}{2}x} - [8x e^{\frac{1}{2}x} - \int 8e^{\frac{1}{2}x}\,dx]$	A1	
		$= 2x^2 e^{\frac{1}{2}x} - 8x e^{\frac{1}{2}x} + 16e^{\frac{1}{2}x} + c$ or $2e^{\frac{1}{2}x}(x^2 - 4x + 8) + c$	A1	
	(ii)	$u = \sin t \Rightarrow \frac{du}{dt} = \cos t$	M1	
		$t = 0 \Rightarrow u = 0,\; t = \frac{\pi}{2} \Rightarrow u = 1$	B1	
		$\sin^2 2t = 4\sin^2 t\cos^2 t = 4\sin^2 t\,(1 - \sin^2 t)$	M1	
		$I = \int_0^1 4u^2(1 - u^2)\,du$		
		$= 4\int_0^1 (u^2 - u^4)\,du$	A1	
		$= 4[\frac{1}{3}u^3 - \frac{1}{5}u^5]_0^1$	M1	
		$= 4[(\frac{1}{3} - \frac{1}{5}) - (0)] = \frac{8}{15}$	M1 A1	**(13)**

Total	**(72)**