C4 Paper I – Marking Guide

1.	*(i)*	$= \frac{1}{\cos x} \times (-\sin x) = -\tan x$	M1 A1	
	(ii)	$= 2x \times \sin 3x + x^2 \times 3\cos 3x = 2x\sin 3x + 3x^2\cos 3x$	M1 A1	**(4)**
2.	*(i)*	$2x + 3y + 3x\frac{dy}{dx} - 4y\frac{dy}{dx} = 0$	M1 A1	
		$\frac{dy}{dx} = \frac{2x+3y}{4y-3x}$	M1 A1	
	(ii)	grad $= \frac{6-6}{-8-9} = 0$	M1	
		$\therefore$ normal parallel to y-axis $\therefore x = 3$	M1 A1	**(7)**
3.	*(i)*	$f(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{(2x+1)(x-3)}$		
		$= \frac{3(2x^2-5x-3)-(x-1)(2x+1)+(x+11)}{(2x+1)(x-3)}$	M1	
		$= \frac{4x^2-13x+3}{(2x+1)(x-3)}$	A1	
		$= \frac{(4x-1)(x-3)}{(2x+1)(x-3)}$	M1	
		$= \frac{4x-1}{2x+1}$	A1	
	(ii)	$(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(2x)^3 + \ldots$	M1	
		$= 1 - 2x + 4x^2 - 8x^3 + \ldots$	A2	
		$\therefore f(x) = (4x-1)(1 - 2x + 4x^2 - 8x^3 + \ldots)$		
		$= 4x - 8x^2 + 16x^3 - 1 + 2x - 4x^2 + 8x^3 + \ldots$	M1	
		$= -1 + 6x - 12x^2 + 24x^3 + \ldots$	A1	**(9)**
4.	*(i)*	$\frac{dx}{dt} = 3t^2, \; \frac{dy}{dt} = -2t^{-2}$	M1	
		$\frac{dy}{dx} = -\frac{2}{3}t^{-4}$	M1 A1	
		$t = 1, \; x = 2, \; y = 2$, grad $= -\frac{2}{3}$, grad of normal $= \frac{3}{2}$	M1	
		$\therefore y - 2 = \frac{3}{2}(x-2)$	M1	
		$y = \frac{3}{2}x - 1$	A1	
	(ii)	$t = \frac{2}{y} \quad \therefore x = (\frac{2}{y})^3 + 1 = \frac{8}{y^3} + 1$	M1	
		$\therefore y^3 = \frac{8}{x-1}, \quad y = \frac{2}{\sqrt[3]{x-1}}$	M1 A1	**(9)**
5.	*(i)*	$15 - 17x \equiv A(1-3x)^2 + B(2+x)(1-3x) + C(2+x)$	M1	
		$x = -2 \Rightarrow 49 = 49A \Rightarrow A = 1$	B1	
		$x = \frac{1}{3} \Rightarrow \frac{28}{3} = \frac{7}{3}C \Rightarrow C = 4$	B1	
		coeffs $x^2 \Rightarrow 0 = 9A - 3B \Rightarrow B = 3$	M1 A1	
	(ii)	$= \int_{-1}^{0} (\frac{1}{2+x} + \frac{3}{1-3x} + \frac{4}{(1-3x)^2})\,dx$		
		$= [\ln\lvert 2+x\rvert - \ln\lvert 1-3x\rvert + \frac{4}{3}(1-3x)^{-1}]_{-1}^{0}$	M1 A2	
		$= (\ln 2 + 0 + \frac{4}{3}) - (0 - \ln 4 + \frac{1}{3})$	M1	
		$= 1 + \ln 8$	A1	**(10)**

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6.	*(i)*	$1 + 3\lambda = -5 \quad \therefore\ \lambda = -2$	M1	
		$p - \lambda = 9 \quad \therefore\ p = 7$	A1	
		$-5 + q\lambda = -9 \quad \therefore\ q = 2$	A1	
	(ii)	$1 + 3\lambda = 25 \quad \therefore\ \lambda = 8$	M1	
		when $\lambda = 8$, $\mathbf{r} = \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix} + 8\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 25 \\ -1 \\ 11 \end{pmatrix}$		
		$\therefore$ (25, −1, 11) lies on l	A1	
	(iii)	$\overrightarrow{OC} = \begin{pmatrix} 1+3\lambda \\ 7-\lambda \\ -5+2\lambda \end{pmatrix} \quad \therefore \begin{pmatrix} 1+3\lambda \\ 7-\lambda \\ -5+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 0$	M1	
		$3 + 9\lambda - 7 + \lambda - 10 + 4\lambda = 0$		
		$\lambda = 1 \quad \therefore\ \overrightarrow{OC} = \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix}$, $C\,(4, 6, -3)$	M1 A1	
	(iv)	$A : \lambda = -2,\ B : \lambda = 8,\ C : \lambda = 1 \quad \therefore\ AC : CB = 3 : 7$	M1 A1	**(10)**
7.	*(i)*	$x = 2\sin u \Rightarrow \frac{dx}{du} = 2\cos u$	M1	
		$x = 0 \Rightarrow u = 0,\ x = \sqrt{3} \Rightarrow u = \frac{\pi}{3}$	B1	
		$I = \int_0^{\frac{\pi}{3}} \frac{1}{2\cos u} \times 2\cos u\ du = \int_0^{\frac{\pi}{3}} 1\ du$	M1 A1	
		$= [u]_0^{\frac{\pi}{3}} = \frac{\pi}{3} - 0 = \frac{\pi}{3}$	M1 A1	
	(ii)	$u = x,\ u' = 1,\ v' = \cos x,\ v = \sin x$	M1	
		$I = [x\sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\ dx$	A1	
		$= [x\sin x + \cos x]_0^{\frac{\pi}{2}}$	M1	
		$= (\frac{\pi}{2} + 0) - (0 + 1) = \frac{\pi}{2} - 1$	M1 A1	**(11)**
8.	*(i)*	$\frac{dN}{dt} = kN$	B1	
	(ii)	$\int \frac{1}{N}\ dN = \int k\ dt$	M1	
		$\ln\lvert N\rvert = kt + c$	M1 A1	
		$t = 0,\ N = N_0 \Rightarrow \ln\lvert N_0\rvert = c$	M1	
		$\ln\lvert N\rvert = kt + \ln\lvert N_0\rvert$		
		$\ln\left\lvert \frac{N}{N_0} \right\rvert = kt$	M1	
		$\frac{N}{N_0} = e^{kt}, \quad N = N_0e^{kt}$	A1	
	(iii)	$2N_0 = N_0e^{6k}$	M1	
		$k = \frac{1}{6}\ln 2 = 0.116$ (3sf)	M1 A1	
	(iv)	$10N_0 = N_0e^{0.1155t}$		
		$t = \frac{1}{0.1155}\ln 10 = 19.932$ hours = 19 hours 56 mins	M1 A1	**(12)**
			Total	**(72)**

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