Core Mathematics 4 Paper H

1. Express

$$\frac{x-10}{(x-3)(x+4)} - \frac{x-8}{(x-3)(2x-1)}$$

as a single fraction in its simplest form. [4]

2. *(i)* Expand $(1+4x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [4]

(ii) State the set of values of x for which your expansion is valid. [1]

3. A curve has the equation

$$3x^2 + xy - 2y^2 + 25 = 0.$$

Find an equation for the normal to the curve at the point with coordinates (1, 4), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [7]

4. The line l_1 passes through the points P and Q with position vectors $(-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k})$ and $(2\mathbf{i} - 9\mathbf{j} + \mathbf{k})$ respectively, relative to a fixed origin.

(i) Find a vector equation for l_1. [2]

The line l_2 has the equation

$$\mathbf{r} = (6\mathbf{i} + a\mathbf{j} + b\mathbf{k}) + t(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$$

and also passes through the point Q.

(ii) Find the values of the constants a and b. [3]

(iii) Find, in degrees to 1 decimal place, the acute angle between lines l_1 and l_2. [4]

5. *(i)* Given that

$$x = \sec \tfrac{y}{2}, \quad 0 \le y < \pi,$$

show that

$$\frac{dy}{dx} = \frac{2}{x\sqrt{x^2 - 1}}. \qquad [4]$$

(ii) Find an equation for the tangent to the curve $y = \sqrt{3 + 2\cos x}$ at the point where $x = \tfrac{\pi}{3}$. [5]

6. A curve has parametric equations

$$x = \frac{t}{2-t}, \quad y = \frac{1}{1+t}, \quad -1 < t < 2.$$

(i) Show that $\dfrac{dy}{dx} = -\tfrac{1}{2}\left(\dfrac{2-t}{1+t}\right)^2$. [4]

(ii) Find an equation for the normal to the curve at the point where $t = 1$. [3]

(iii) Show that the cartesian equation of the curve can be written in the form

$$y = \frac{1+x}{1+3x}. \qquad [4]$$

7. *(i)* Find

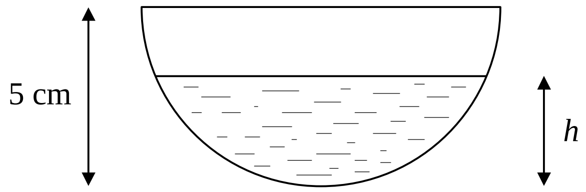
$$\int x^2 \sin x \ dx. \qquad [5]$$

(ii) Use the substitution $u = 1 + \sin x$ to find the value of

$$\int_0^{\frac{\pi}{2}} \cos x \,(1 + \sin x)^3 \ dx. \qquad [6]$$

Turn over

8.



The diagram shows a hemispherical bowl of radius 5 cm.

The bowl is filled with water but the water leaks from a hole at the base of the bowl. At time t minutes, the depth of water is h cm and the volume of water in the bowl is V cm^3, where

$$V = \tfrac{1}{3}\pi h^2(15 - h).$$

In a model it is assumed that the rate at which the volume of water in the bowl decreases is proportional to V.

(i) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{kh(15-h)}{3(10-h)},$$

where k is a positive constant. [5]

(ii) Express $\dfrac{3(10-h)}{h(15-h)}$ in partial fractions. [3]

Given that when $t = 0$, $h = 5$,

(iii) show that

$$h^2(15 - h) = 250\,\mathrm{e}^{-kt}.$$ [6]

Given also that when $t = 2$, $h = 4$,

(iv) find the value of k to 3 significant figures. [2]