C4 Paper H – Marking Guide

1. $= \dfrac{(x-10)(2x-1)-(x-8)(x+4)}{(x-3)(x+4)(2x-1)}$ M1

$= \dfrac{x^2-17x+42}{(x-3)(x+4)(2x-1)}$ A1

$= \dfrac{(x-14)(x-3)}{(x-3)(x+4)(2x-1)}$ M1

$= \dfrac{x-14}{(x+4)(2x-1)}$ A1 **(4)**

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2. *(i)* $= 1 + (\frac{3}{2})(4x) + \frac{(\frac{3}{2})(\frac{1}{2})}{2}(4x)^2 + \frac{(\frac{3}{2})(\frac{1}{2})(-\frac{1}{2})}{3\times 2}(4x)^3 + \ldots$ M1

$= 1 + 6x + 6x^2 - 4x^3 + \ldots$ A3

(ii) $|x| < \frac{1}{4}$ B1 **(5)**

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3. $6x + y + x\dfrac{dy}{dx} - 4y\dfrac{dy}{dx} = 0$ M1 A1

$(1, 4) \Rightarrow 6 + 4 + \dfrac{dy}{dx} - 16\dfrac{dy}{dx} = 0, \quad \dfrac{dy}{dx} = \frac{2}{3}$ M1 A1

grad of normal $= -\frac{3}{2}$ M1

$\therefore\ y - 4 = -\frac{3}{2}(x-1)$ M1

$2y - 8 = -3x + 3$

$3x + 2y - 11 = 0$ A1 **(7)**

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4. *(i)* $\overrightarrow{PQ} = (2\mathbf{i} - 9\mathbf{j} + \mathbf{k}) - (-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}) = (3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ M1

$\therefore\ \mathbf{r} = (-\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}) + s(3\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ A1

(ii) $6 + t = 2 \quad \therefore t = -4$ M1

$a + 4t = -9 \quad \therefore a = 7$ A1

$b - t = 1 \quad \therefore b = -3$ A1

(iii) $= \cos^{-1}\left|\dfrac{3\times1+(-1)\times4+(-2)\times(-1)}{\sqrt{9+1+4}\times\sqrt{1+16+1}}\right|$ M1 A1

$= \cos^{-1}\dfrac{1}{\sqrt{14}\times\sqrt{18}} = 86.4°$ (1dp) M1 A1 **(9)**

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5. *(i)* $\dfrac{dx}{dy} = \frac{1}{2}\sec\frac{y}{2}\tan\frac{y}{2}$ M1

$0 \le y < \pi \ \therefore \tan\frac{y}{2} \ge 0 \ \therefore \dfrac{dx}{dy} = \frac{1}{2}\sec\frac{y}{2}\sqrt{\sec^2\frac{y}{2}-1} = \frac{1}{2}x\sqrt{x^2-1}$ M1

$\dfrac{dy}{dx} = 1 \div \dfrac{dx}{dy} = \dfrac{2}{x\sqrt{x^2-1}}$ M1 A1

(ii) $\dfrac{dy}{dx} = \frac{1}{2}(3+2\cos x)^{-\frac{1}{2}} \times (-2\sin x) = -\dfrac{\sin x}{\sqrt{3+2\cos x}}$ M1 A1

$x = \frac{\pi}{3},\ y = 2,\ \text{grad} = -\frac{1}{4}\sqrt{3}$ B1

$\therefore\ y - 2 = -\frac{1}{4}\sqrt{3}(x - \frac{\pi}{3}) \qquad [\ 3\sqrt{3}x + 12y - 24 - \pi\sqrt{3} = 0\ ]$ M1 A1 **(9)**

6.	*(i)*	$\frac{dx}{dt} = \frac{1 \times (2-t) - t \times (-1)}{(2-t)^2} = \frac{2}{(2-t)^2}$, $\frac{dy}{dt} = -(1+t)^{-2}$	M1 B1	
		$\frac{dy}{dx} = -\frac{1}{(1+t)^2} \div \frac{2}{(2-t)^2} = -\frac{(2-t)^2}{2(1+t)^2} = -\frac{1}{2}\left(\frac{2-t}{1+t}\right)^2$	M1 A1	
	(ii)	$t = 1,\ x = 1,\ y = \frac{1}{2}$, grad $= -\frac{1}{8}$	B1	
		grad of normal = 8		
		$\therefore\ y - \frac{1}{2} = 8(x-1)$ $\quad [\ y = 8x - \frac{15}{2}\ ]$	M1 A1	
	(iii)	$x(2-t) = t$	M1	
		$2x = t(1+x),\ t = \frac{2x}{1+x}$	A1	
		$y = \frac{1}{1+\frac{2x}{1+x}} = \frac{1+x}{(1+x)+2x}$ $\quad \therefore\ y = \frac{1+x}{1+3x}$	M1 A1	**(11)**
7.	*(i)*	$u = x^2,\ u' = 2x,\ v' = \sin x,\ v = -\cos x$	M1	
		$I = -x^2 \cos x - \int -2x \cos x\ dx = -x^2 \cos x + \int 2x \cos x\ dx$	A1	
		$u = 2x,\ u' = 2,\ v' = \cos x,\ v = \sin x$	M1	
		$I = -x^2 \cos x + 2x \sin x - \int 2 \sin x\ dx$	A1	
		$= -x^2 \cos x + 2x \sin x + 2 \cos x + c$	A1	
	(ii)	$u = 1 + \sin x \Rightarrow \frac{du}{dx} = \cos x$	M1	
		$x = 0 \Rightarrow u = 1,\ x = \frac{\pi}{2} \Rightarrow u = 2$	B1	
		$I = \int_1^2 u^3\ du$	M1 A1	
		$= [\frac{1}{4}u^4]_1^2$	M1	
		$= 4 - \frac{1}{4} = \frac{15}{4}$	A1	**(11)**
8.	*(i)*	$\frac{dV}{dt} = -kV,\ \frac{dV}{dh} = 10\pi h - \pi h^2$	B2	
		$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $\quad \therefore\ -kV = (10\pi h - \pi h^2)\frac{dh}{dt}$	M1	
		$-\frac{1}{3}k\pi h^2(15-h) = \pi h(10-h)\frac{dh}{dt}$		
		$-kh(15-h) = 3(10-h)\frac{dh}{dt}$ $\quad \therefore\ \frac{dh}{dt} = -\frac{kh(15-h)}{3(10-h)}$	M1 A1	
	(ii)	$\frac{3(10-h)}{h(15-h)} \equiv \frac{A}{h} + \frac{B}{15-h}$, $\quad 3(10-h) \equiv A(15-h) + Bh$	M1	
		$h = 0 \Rightarrow A = 2,\ h = 15 \Rightarrow B = -1$ $\quad \therefore\ \frac{3(10-h)}{h(15-h)} \equiv \frac{2}{h} - \frac{1}{15-h}$	A2	
	(iii)	$\int \frac{3(10-h)}{h(15-h)}\ dh = \int -k\ dt$, $\quad \int (\frac{2}{h} - \frac{1}{15-h})\ dh = \int -k\ dt$	M1	
		$2\ln\lvert h\rvert + \ln\lvert 15-h\rvert = -kt + c$	M1 A1	
		$t = 0,\ h = 5 \Rightarrow 2\ln 5 + \ln 10 = c,\ \ c = \ln 250$	M1	
		$2\ln\lvert h\rvert + \ln\lvert 15-h\rvert - \ln 250 = -kt$		
		$\ln\frac{h^2(15-h)}{250} = -kt,\ \frac{h^2(15-h)}{250} = e^{-kt},\ \ h^2(15-h) = 250e^{-kt}$	M1 A1	
	(iv)	$t = 2,\ h = 4 \Rightarrow 176 = 250e^{-2k}$		
		$k = -\frac{1}{2}\ln\frac{176}{250} = 0.175$ (3sf)	M1 A1	**(16)**
			Total	**(72)**