Core Mathematics 4 Paper G

1. Express

$$\frac{2x^3 + x^2}{x^2 - 4} \times \frac{x - 2}{2x^2 - 5x - 3}$$

as a single fraction in its simplest form. [4]

2. A curve has the equation

$$x^3 + 2xy - y^2 + 24 = 0.$$

Show that the normal to the curve at the point (2, –4) has the equation $y = 3x - 10$. [7]

3. Using the substitution $u = e^x - 1$, show that

$$\int_{\ln 2}^{\ln 5} \frac{e^{2x}}{\sqrt{e^x - 1}}\, dx = \tfrac{20}{3}.$$ [8]

4. *(i)* Expand $(1 + ax)^{-3}$, $|ax| < 1$, in ascending powers of x up to and including the term in x^3. Give each coefficient as simply as possible in terms of the constant a. [3]

Given that the coefficient of x^2 in the expansion of $\frac{6 - x}{(1 + ax)^3}$, $|ax| < 1$, is 3,

(ii) find the two possible values of a. [4]

Given also that $a < 0$,

(iii) show that the coefficient of x^3 in the expansion of $\frac{6 - x}{(1 + ax)^3}$ is $\frac{14}{9}$. [2]

5. $$\text{f}(x) = \frac{7 + 3x + 2x^2}{(1 - 2x)(1 + x)^2}, \quad |x| > \tfrac{1}{2}.$$

(i) Express f(x) in partial fractions. [5]

(ii) Show that

$$\int_1^2 \text{f}(x)\, dx = p - \ln q,$$

where p is rational and q is an integer. [5]

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6. Relative to a fixed origin, the points A, B and C have position vectors $(2\mathbf{i} - \mathbf{j} + 6\mathbf{k})$, $(5\mathbf{i} - 4\mathbf{j})$ and $(7\mathbf{i} - 6\mathbf{j} - 4\mathbf{k})$ respectively.

(i) Show that A, B and C all lie on a single straight line. [3]

(ii) Write down the ratio $AB : BC$ [1]

The point D has position vector $(3\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

(iii) Show that AD is perpendicular to BD. [4]

(iv) Find the exact area of triangle ABD. [3]

7. A mathematician is selling goods at a car boot sale. She believes that the rate at which she makes sales depends on the length of time since the start of the sale, t hours, and the total value of sales she has made up to that time, £ x.

She uses the model

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k(5-t)}{x},$$

where k is a constant.

Given that after two hours she has made sales of £96 in total,

(i) solve the differential equation and show that she made £72 in the first hour of the sale. [7]

The mathematician believes that is it not worth staying at the sale once she is making sales at a rate of less than £10 per hour.

(ii) Verify that at 3 hours and 5 minutes after the start of the sale, she should have already left. [4]

Turn over

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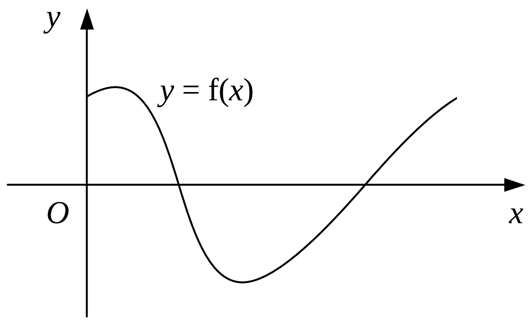
8.



The diagram shows the curve $y = \mathrm{f}(x)$ in the interval $0 \le x \le 2\pi$ where

$$\mathrm{f}(x) = \frac{\cos x}{2 - \sin x}, \quad x \in \mathbb{R}.$$

(i) Show that $\mathrm{f}'(x) = \dfrac{1 - 2\sin x}{(2 - \sin x)^2}$. [3]

(ii) Find an equation for the tangent to the curve $y = \mathrm{f}(x)$ at the point where $x = \pi$. [3]

(iii) Find the minimum and maximum values of $\mathrm{f}(x)$ in the interval $0 \le x \le 2\pi$. [4]

(iv) Explain why your answers to part *(c)* are the minimum and maximum values of $\mathrm{f}(x)$ for all real values of x. [2]

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