C4 Paper G – Marking Guide

1. $= \dfrac{x^2(2x+1)}{(x+2)(x-2)} \times \dfrac{x-2}{(2x+1)(x-3)}$ M1 A1

$= \dfrac{x^2}{(x+2)(x-3)}$ M1 A1 **(4)**

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2. $3x^2 + 2y + 2x\dfrac{dy}{dx} - 2y\dfrac{dy}{dx} = 0$ M1 A1

$(2, -4) \Rightarrow 12 - 8 + 4\dfrac{dy}{dx} + 8\dfrac{dy}{dx} = 0, \quad \dfrac{dy}{dx} = -\frac{1}{3}$ M1 A1

grad of normal = 3 M1

$\therefore\ y + 4 = 3(x - 2)$ M1

$y = 3x - 10$ A1 **(7)**

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3. $u = e^x - 1 \Rightarrow \dfrac{du}{dx} = e^x = u + 1$ M1

$x = \ln 2 \Rightarrow u = 1,\ x = \ln 5 \Rightarrow u = 4$ B1

$I = \displaystyle\int_1^4 \frac{(u+1)^2}{\sqrt{u}} \times \frac{1}{u+1}\ du$ M1

$= \displaystyle\int_1^4 (u^{\frac{1}{2}} + u^{-\frac{1}{2}})\ du$ A1

$= [\frac{2}{3}u^{\frac{3}{2}} + 2u^{\frac{1}{2}}]_1^4$ M1 A1

$= (\frac{16}{3} + 4) - (\frac{2}{3} + 2) = \frac{20}{3}$ M1 A1 **(8)**

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4. *(i)* $= 1 + (-3)(ax) + \frac{(-3)(-4)}{2}(ax)^2 + \frac{(-3)(-4)(-5)}{3\times 2}(ax)^3 + \ldots$ M1 A1

$= 1 - 3ax + 6a^2x^2 - 10a^3x^3 + \ldots$ A1

(ii) $\dfrac{6-x}{(1+ax)^3} = (6 - x)(1 - 3ax + 6a^2x^2 + \ldots)$

coeff. of $x^2 = 36a^2 + 3a = 3$ M1

$12a^2 + a - 1 = 0$ A1

$(4a - 1)(3a + 1) = 0$ M1

$a = -\frac{1}{3}, \frac{1}{4}$ A1

(iii) $a = -\frac{1}{3}\ \therefore\ \dfrac{6-x}{(1+ax)^3} = (6 - x)(\ldots + \frac{2}{3}x^2 + \frac{10}{27}x^3 + \ldots)$ M1

coeff. of $x^3 = (6 \times \frac{10}{27}) + (-1 \times \frac{2}{3}) = \frac{20}{9} - \frac{2}{3} = \frac{14}{9}$ A1 **(9)**

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5. *(i)* $\dfrac{7+3x+2x^2}{(1-2x)(1+x)^2} \equiv \dfrac{A}{1-2x} + \dfrac{B}{1+x} + \dfrac{C}{(1+x)^2}$

$7 + 3x + 2x^2 \equiv A(1 + x)^2 + B(1 - 2x)(1 + x) + C(1 - 2x)$ M1

$x = \frac{1}{2} \Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4$ B1

$x = -1 \Rightarrow 6 = 3C \Rightarrow C = 2$ B1

coeffs $x^2 \Rightarrow 2 = A - 2B \Rightarrow B = 1$ M1 A1

$\therefore\ f(x) = \dfrac{4}{1-2x} + \dfrac{1}{1+x} + \dfrac{2}{(1+x)^2}$

(ii) $= \displaystyle\int_1^2 \left(\frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2}\right) dx$

$= [-2\ln|1 - 2x| + \ln|1 + x| - 2(1 + x)^{-1}]_1^2$ M1 A2

$= (-2\ln 3 + \ln 3 - \frac{2}{3}) - (0 + \ln 2 - 1)$ M1

$= -\ln 3 - \ln 2 + \frac{1}{3} = \frac{1}{3} - \ln 6 \quad [\,p = \frac{1}{3}, q = 6\,]$ A1 **(10)**

6.	*(i)*	$\overrightarrow{AB} = (5\mathbf{i} - 4\mathbf{j}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (3\mathbf{i} - 3\mathbf{j} - 6\mathbf{k})$	B1	
		$\overrightarrow{AC} = (7\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (5\mathbf{i} - 5\mathbf{j} - 10\mathbf{k}) = \frac{5}{3}\overrightarrow{AB}$	M1	
		$\therefore$ $\overrightarrow{AC}$ is parallel to $\overrightarrow{AB}$, also common point $\therefore$ single straight line	A1	
	(ii)	3 : 2	B1	
	(iii)	$\overrightarrow{AD} = (3\mathbf{i} + \mathbf{j} + 4\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) = (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	B1	
		$\overrightarrow{BD} = (3\mathbf{i} + \mathbf{j} + 4\mathbf{k}) - (5\mathbf{i} - 4\mathbf{j}) = (-2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$	B1	
		$\overrightarrow{AD} . \overrightarrow{BD} = -2 + 10 - 8 = 0$ $\therefore$ perpendicular	M1 A1	
	(iv)	$= \frac{1}{2} \times \sqrt{1+4+4} \times \sqrt{4+25+16} = \frac{1}{2} \times 3 \times 3\sqrt{5} = \frac{9}{2}\sqrt{5}$	M2 A1	**(11)**

7.	*(i)*	$\int x \; \mathrm{d}x = \int k(5 - t) \; \mathrm{d}t$	M1	
		$\frac{1}{2}x^2 = k(5t - \frac{1}{2}t^2) + c$	M1 A1	
		$t = 0,\; x = 0 \Rightarrow c = 0$	B1	
		$t = 2,\; x = 96 \Rightarrow 4608 = 8k,\quad k = 576$	M1	
		$t = 1 \Rightarrow \frac{1}{2}x^2 = 576 \times \frac{9}{2},\quad x = \sqrt{5184} = 72$	M1 A1	
	(ii)	3 hours 5 mins $\Rightarrow t = 3.0833,\; x = \sqrt{12284} = 110.83$	M1 A1	
		$\therefore \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{576(5-3.0833)}{110.83} = 9.96,\quad \frac{\mathrm{d}x}{\mathrm{d}t} < 10$ so she should have left	M1 A1	**(11)**

8.	*(i)*	$\mathrm{f}'(x) = \frac{(-\sin x)\times(2-\sin x) - \cos x \times(-\cos x)}{(2-\sin x)^2}$	M1 A1	
		$= \frac{-2\sin x + \sin^2 x + \cos^2 x}{(2-\sin x)^2}$		
		$= \frac{1-2\sin x}{(2-\sin x)^2}$	A1	
	(ii)	$x = \pi,\; y = -\frac{1}{2},\;$ grad $= \frac{1}{4}$	B1	
		$\therefore\; y + \frac{1}{2} = \frac{1}{4}(x - \pi)\qquad [\, x - 4y - 2 - \pi = 0 \,]$	M1 A1	
	(iii)	from graph, min. and max. values at SP		
		SP: $\frac{1-2\sin x}{(2-\sin x)^2} = 0$		
		$\sin x = \frac{1}{2}$	M1	
		$x = \frac{\pi}{6},\; \pi - \frac{\pi}{6} = \frac{\pi}{6},\; \frac{5\pi}{6}$	A1	
		at SP, $y = \frac{\pm\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \pm\frac{1}{3}\sqrt{3}$	M1	
		$\therefore$ min. $= -\frac{1}{3}\sqrt{3}$, max. $= \frac{1}{3}\sqrt{3}$	A1	
	(iv)	$\sin x$ and $\cos x$ both have period 2π $\mathrm{f}(x)$ is a function of $\sin x$ and $\cos x$ and $\therefore$ also has period 2π $\therefore$ values of $\mathrm{f}(x)$ in interval $0 \le x \le 2\pi$ are just repeated	B2	**(12)**

Total **(72)**