C4 Paper F – Marking Guide

1.

$$x^2 - 3x + 3 \overline{)\, x^4 + x^3 - 13x^2 + 26x - 17}$$

$$\begin{array}{r} x^2 + 4x - 4 \\ \hline x^4 - 3x^3 + 3x^2 \\ \hline 4x^3 - 16x^2 + 26x \\ 4x^3 - 12x^2 + 12x \\ \hline -4x^2 + 14x - 17 \\ -4x^2 + 12x - 12 \\ \hline 2x - 5 \end{array}$$

M2

$\therefore\ f(x) = x^2 + 4x - 4 + \frac{2x-5}{x^2-3x+3}$, $A = 4$, $B = -4$, $C = 2$, $D = -5$ A2 **(4)**

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2. $u = 1 - x^{\frac{1}{2}} \Rightarrow x = (1-u)^2$, $\frac{dx}{du} = -2(1-u) = 2u - 2$ M1 A1

$I = \int \frac{1}{u} \times (2u - 2)\ du = \int (2 - \frac{2}{u})\ du$ A1

$= 2u - 2\ln|u| + c$ M1 A1

$= 2(1 - x^{\frac{1}{2}}) - 2\ln|1 - x^{\frac{1}{2}}| + c$ A1 **(6)**

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3. *(i)* $-4\sin x + (2\cos y)\frac{dy}{dx} = 0$ M1 A1

$\frac{dy}{dx} = \frac{4\sin x}{2\cos y} = \frac{2\sin x}{\cos y} = 2\sin x \sec y$ M1 A1

(ii) grad $= 2 \times \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = 2$ B1

$\therefore\ y - \frac{\pi}{6} = 2(x - \frac{\pi}{3})$ M1

$6y - \pi = 12x - 4\pi$

$4x - 2y = \pi$ A1 **(7)**

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4. *(i)* $\frac{3x+6}{x(3-x)} \equiv \frac{A}{x} + \frac{B}{3-x}$

$3x + 6 \equiv A(3 - x) + Bx$ M1

$x = 0 \Rightarrow 6 = 3A \Rightarrow A = 2$ A1

$x = 3 \Rightarrow 15 = 3B \Rightarrow B = 5$ A1

$\therefore\ \frac{3x+6}{3x-x^2} \equiv \frac{2}{x} + \frac{5}{3-x}$

(ii) $= \int_1^2 (\frac{2}{x} + \frac{5}{3-x})\ dx$

$= [2\ln|x| - 5\ln|3 - x|]_1^2$ M1 A1

$= (2\ln 2 - 0) - (0 - 5\ln 2) = 7\ln 2$ M1 A1 **(7)**

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5. volume $= \pi\int_0^2 16x\, e^{-2x}\ dx$ M1

$u = 16x$, $u' = 16$, $v' = e^{-2x}$, $v = -\frac{1}{2}e^{-2x}$ M1

$I = -8x\, e^{-2x} - \int -8e^{-2x}\ dx$ A1

$= -8x\, e^{-2x} - 4e^{-2x} + c$ M1 A1

volume $= \pi[-8x\, e^{-2x} - 4e^{-2x}]_0^2$

$= \pi\{(-16e^{-4} - 4e^{-4}) - (0 - 4)\}$ M1

$= 4\pi(1 - 5e^{-4})$ A1 **(7)**

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6. *(i)* $f(\frac{1}{10}) = \frac{3}{\sqrt{1-\frac{1}{10}}} = \frac{3}{\sqrt{\frac{9}{10}}} = \frac{3}{(\frac{3}{\sqrt{10}})} = \sqrt{10}$ M1 A1

(ii) $= 3(1-x)^{-\frac{1}{2}} = 3[1 + (-\frac{1}{2})(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(-x)^3 + \ldots]$ M1

$= 3 + \frac{3}{2}x + \frac{9}{8}x^2 + \frac{15}{16}x^3 + \ldots$ A3

(iii) $\sqrt{10} = f(\frac{1}{10}) \approx 3 + \frac{3}{20} + \frac{9}{800} + \frac{15}{16000} = 3.1621875$ (8sf) B1

(iv) $= \frac{\sqrt{10} - 3.1621875}{\sqrt{10}} \times 100\% = 0.003\%$ (1sf) M1 A1 **(9)**

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7. *(i)* $4s = 6 + 14t$ (1)

$-3 - 2s = 3 + 2t$ (2) B1

(1) + 2 × (2): $-6 = 12 + 18t$, $t = -1$, $s = -2$ M1 A1

$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix} - 2\begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \\ 1 \end{pmatrix}$ A1

(ii) $a - (-5) = -3$, $a = -8$ M1 A1

(iii) $\cos\theta = \left| \frac{5\times(-5) + 4\times 14 + (-2)\times 2}{\sqrt{25+16+4}\times\sqrt{25+196+4}} \right|$ M1 A1

$= \frac{27}{\sqrt{45}\times 15} = \frac{9}{3\sqrt{5}\times 5} = \frac{3}{5\sqrt{5}} = \frac{3}{25}\sqrt{5}$ M1 A1 **(10)**

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8. *(i)* $\int \frac{1}{P}\,dP = \int 0.05e^{-0.05t}\,dt$ M1

$\ln|P| = -e^{-0.05t} + c$ M1 A1

$t = 0$, $P = 9000 \Rightarrow \ln 9000 = -1 + c$, $c = 1 + \ln 9000$ M1

$\ln|P| = 1 + \ln 9000 - e^{-0.05t}$ A1

$t = 10 \Rightarrow \ln|P| = 1 + \ln 9000 - e^{-0.5} = 9.498$ M1

$P = e^{9.498} = 13339 = 13300$ (3sf) A1

(ii) $t \to \infty$, $\ln|P| \to 1 + \ln 9000$ M1

$\therefore P \to e^{1 + \ln 9000} = 9000e = 24465 = 24500$ (3sf) M1 A1 **(10)**

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9. *(i)* $\frac{dx}{dt} = 2t - 1$, $\frac{dy}{dt} = \frac{4\times(1-t) - 4t\times(-1)}{(1-t)^2} = \frac{4}{(1-t)^2}$ M1

$\frac{dy}{dx} = \frac{4}{(2t-1)(1-t)^2}$ M1 A1

(ii) $t = -1$, $x = 2$, $y = -2$, grad $= -\frac{1}{3}$ M1

$\therefore y + 2 = -\frac{1}{3}(x - 2)$ M1

$3y + 6 = -x + 2$

$x + 3y + 4 = 0$ A1

(iii) $t(t-1) + 3\times\frac{4t}{1-t} + 4 = 0$ M1

$-t(t-1)^2 + 12t + 4(1-t) = 0$

$t^3 - 2t^2 - 7t - 4 = 0$ A1

$t = -1$ is a solution $\therefore (t+1)$ is a factor M1

$(t+1)(t^2 - 3t - 4) = 0$ M1

$(t+1)(t+1)(t-4) = 0$

$t = -1$ (at P) or $t = 4$ $\therefore Q(12, -\frac{16}{3})$ M1 A1 **(12)**

Total **(72)**