C4 Paper E – Marking Guide

1.		$\text{f}(x) = 1 + \frac{4x}{2x-5} - \frac{15}{(2x-5)(x-1)}$		
		$= \frac{2x^2 - 7x + 5 + 4x(x-1) - 15}{(2x-5)(x-1)}$	M1	
		$= \frac{6x^2 - 11x - 10}{(2x-5)(x-1)}$	A1	
		$= \frac{(3x+2)(2x-5)}{(2x-5)(x-1)}$	M1	
		$= \frac{3x+2}{x-1}$	A1	**(4)**
2.	*(i)*	$2x - 3y - 3x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	M1 A1	
		$\frac{dy}{dx} = \frac{2x-3y}{3x+2y}$	M1 A1	
	(ii)	grad = 5	M1	
		$\therefore\ y + 2 = 5(x - 2)$ [$y = 5x - 12$]	M1 A1	**(7)**
3.	*(i)*	$= -\frac{1}{2}\ln\lvert 2 - x^2\rvert + c$	M1 A2	
	(ii)	$u = x^2,\ u' = 2x,\ v' = e^{-x},\ v = -e^{-x}$	M1	
		$I = -x^2 e^{-x} - \int -2x\,e^{-x}\ dx = -x^2 e^{-x} + \int 2x\,e^{-x}\ dx$	A1	
		$u = 2x,\ u' = 2,\ v' = e^{-x},\ v = -e^{-x}$	M1	
		$I = -x^2 e^{-x} - 2x\,e^{-x} - \int -2e^{-x}\ dx$	A1	
		$= -x^2 e^{-x} - 2x\,e^{-x} - 2e^{-x} + c$	A1	**(8)**
4.	*(i)*	$y = \cos^2\theta = \frac{1}{\sec^2\theta} = \frac{1}{1+\tan^2\theta}\quad \therefore y = \frac{1}{1+x^2}$	M2 A1	
	(ii)	area $= \int_{-1}^{1} \frac{1}{1+x^2}\ dx$		
		$x = \tan u,\ \frac{dx}{du} = \sec^2 u$	M1	
		$x = -1 \Rightarrow u = -\frac{\pi}{4},\ x = 1 \Rightarrow u = \frac{\pi}{4}$	B1	
		area $= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\tan^2 u} \times \sec^2 u\ du$		
		$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{\sec^2 u} \times \sec^2 u\ du = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} du$	A1	
		$= [u]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{4} - (-\frac{\pi}{4}) = \frac{\pi}{2}$	M1 A1	**(8)**
5.	*(i)*	$= 4^{\frac{1}{2}}(1 - \frac{1}{4}x)^{\frac{1}{2}} = 2(1 - \frac{1}{4}x)^{\frac{1}{2}}$	B1	
		$= 2[1 + (\frac{1}{2})(-\frac{1}{4}x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-\frac{1}{4}x)^2 + \ldots] = 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \ldots$	M1 A2	
	(ii)	$\lvert x\rvert < 4$	B1	
	(iii)	$x = 0.01 \Rightarrow (4 - x)^{\frac{1}{2}} = \sqrt{3.99} = \sqrt{\frac{399}{100}} = \frac{1}{10}\sqrt{399}$	M1	
		$x = 0.01 \Rightarrow (4 - x)^{\frac{1}{2}} \approx 2 - \frac{1}{400} - \frac{1}{640000} = 1.997498438$	M1	
		$\therefore\ \sqrt{399} \approx 10 \times 1.997498438 = 19.9749844$ (9sf)	M1 A1	**(9)**

6.	*(i)*	$\frac{d}{dx}(\sec x) = \frac{d}{dx}[(\cos x)^{-1}]$		
		$= -(\cos x)^{-2} \times (-\sin x)$	M1 A1	
		$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$	M1	
		$= \sec x \tan x$	A1	
	(ii)	$\frac{dy}{dx} = 2e^{2x} \times \sec x + e^{2x} \times \sec x \tan x = e^{2x} \sec x\,(2 + \tan x)$	M1 A1	
		$x = 0,\ y = 1,\ \text{grad} = 2$	M1	
		$\therefore\ y = 2x + 1$	A1	
	(iii)	SP: $e^{2x} \sec x\,(2 + \tan x) = 0$	M1	
		$\tan x = -2$	M1	
		$x = -1.11$ (2dp)	A1	**(11)**

7.	*(i)*	$\overrightarrow{AB} = (8\mathbf{j} - 6\mathbf{k}) - (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) = (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	M1	
		$\therefore\ \mathbf{r} = (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) + \lambda(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	A1	
	(ii)	$3 - 3\lambda = -2 + 7\mu$ (1)		
		$6 + 2\lambda = 10 - 4\mu$ (2)		
		$-8 + 2\lambda = 6 + 6\mu$ (3)	B1	
		(3) – (2): $-14 = -4 + 10\mu,\ \mu = -1,\ \lambda = 4$	M1 A1	
		check (1) $3 - 12 = -2 - 7$, true $\therefore$ intersect	B1	
	(iii)	$\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) - (7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$ $\therefore (-9, 14, 0)$	B1	
	(iv)	$\overrightarrow{OC} = [(-2 + 7\mu)\mathbf{i} + (10 - 4\mu)\mathbf{j} + (6 + 6\mu)\mathbf{k}]$		
		$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}]$	M1 A1	
		$\therefore\ [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}].(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0$	M1	
		$15 - 21\mu + 8 - 8\mu + 28 + 12\mu = 0$		
		$\mu = 3\ \therefore\ \overrightarrow{OC} = (19\mathbf{i} - 2\mathbf{j} + 24\mathbf{k})$	M1 A1	**(12)**

8.	*(i)*	when $x = \frac{1}{4},\ \frac{dx}{dt} = \frac{3}{4} \div 6 = \frac{1}{8}$	M1	
		$\frac{dx}{dt} = kx(1 - x)\ \ \therefore\ \frac{1}{8} = k \times \frac{1}{4} \times \frac{3}{4},\ k = \frac{2}{3}\ \ \therefore\ \frac{dx}{dt} = \frac{2}{3}x(1 - x)$	M1 A1	
	(ii)	$\int \frac{1}{x(1-x)}\,dx = \int \frac{2}{3}\,dt$	M1	
		$\frac{1}{x(1-x)} \equiv \frac{A}{x} + \frac{B}{1-x},\quad 1 \equiv A(1 - x) + Bx$	M1	
		$x = 0 \Rightarrow A = 1$	A1	
		$x = 1 \Rightarrow B = 1$	A1	
		$\therefore\ \int (\frac{1}{x} + \frac{1}{1-x})\,dx = \int \frac{2}{3}\,dt$		
		$\ln\lvert x\rvert - \ln\lvert 1 - x\rvert = \frac{2}{3}t + c$	M1 A1	
		$t = 0,\ x = \frac{1}{4} \Rightarrow \ln\frac{1}{4} - \ln\frac{3}{4} = c,\quad c = \ln\frac{1}{3}$	M1	
		$t = 3 \Rightarrow \ln\lvert x\rvert - \ln\lvert 1 - x\rvert = 2 + \ln\frac{1}{3}$		
		$\ln\left\lvert\frac{3x}{1-x}\right\rvert = 2,\quad \frac{3x}{1-x} = e^2$	M1	
		$3x = e^2(1 - x),\quad x(e^2 + 3) = e^2$	M1	
		$x = \frac{e^2}{e^2+3}\ \ \therefore$ % destroyed $= \frac{e^2}{e^2+3} \times 100\% = 71.1\%$ (3sf)	A1	**(13)**

Total **(72)**