Core Mathematics 4 Paper D

1. Evaluate

$$\int_0^{\pi} \sin x\,(1 + \cos x)\ \mathrm{d}x. \qquad [4]$$

2. *(i)* Simplify

$$\frac{x^2+7x+12}{2x^2+9x+4}. \qquad [2]$$

(ii) Express

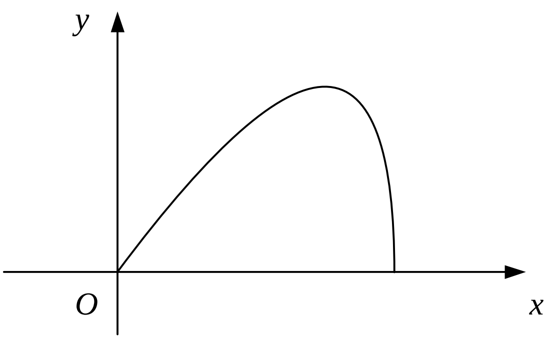
$$\frac{x+4}{2x^2+3x+1} - \frac{2}{2x+1}$$

as a single fraction in its simplest form. [3]

3. Find the exact value of

$$\int_1^{3} x^2 \ln x\ \mathrm{d}x. \qquad [5]$$

4.



The diagram shows the curve with parametric equations

$$x = t + \sin t,\quad y = \sin t,\quad 0 \le t \le \pi.$$

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [3]

(ii) Find, in exact form, the coordinates of the point where the tangent to the curve is parallel to the x-axis. [3]

5. Given that $y = -2$ when $x = 1$, solve the differential equation

$$\frac{dy}{dx} = y^2\sqrt{x},$$

giving your answer in the form $y = f(x)$. [7]

6. *(i)* Find $\int \tan^2 3x \, dx$. [3]

(ii) Using the substitution $u = x^2 + 4$, evaluate

$$\int_0^2 \frac{5x}{(x^2+4)^2} \, dx.$$ [6]

7. A curve has the equation

$$3x^2 - 2x + xy + y^2 - 11 = 0.$$

The point P on the curve has coordinates $(-1, 3)$.

(i) Show that the normal to the curve at P has the equation $y = 2 - x$. [6]

(ii) Find the coordinates of the point where the normal to the curve at P meets the curve again. [4]

8. The line l_1 passes through the points A and B with position vectors $(-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ and $(7\mathbf{i} - \mathbf{j} + 12\mathbf{k})$ respectively, relative to a fixed origin.

(i) Find a vector equation for l_1. [2]

The line l_2 has the equation

$$\mathbf{r} = (5\mathbf{j} - 7\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}).$$

The point C lies on l_2 and is such that AC is perpendicular to BC.

(ii) Show that one possible position vector for C is $(\mathbf{i} + 3\mathbf{j})$ and find the other. [8]

Assuming that C has position vector $(\mathbf{i} + 3\mathbf{j})$,

(iii) find the area of triangle ABC, giving your answer in the form $k\sqrt{5}$. [3]

Turn over

9. $$f(x) = \frac{8-x}{(1+x)(2-x)}, \quad |x| < 1.$$

(i) Express f(x) in partial fractions. [3]

(ii) Show that

$$\int_0^{\frac{1}{2}} f(x)\ dx = \ln k,$$

where k is an integer to be found. [4]

(iii) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [6]