C4 Paper D – Marking Guide

1.		$= [-\frac{1}{2}(1 + \cos x)^2]_0^{\pi}$	M1 A1	
		$= -\frac{1}{2}(0 - 4) = 2$	M1 A1	**(4)**
2.	*(i)*	$= \frac{(x+3)(x+4)}{(2x+1)(x+4)} = \frac{x+3}{2x+1}$	M1 A1	
	(ii)	$= \frac{x+4}{(2x+1)(x+1)} - \frac{2}{2x+1}$	M1	
		$= \frac{(x+4)-2(x+1)}{(2x+1)(x+1)}$	M1	
		$= \frac{2-x}{(2x+1)(x+1)}$	A1	**(5)**
3.		$u = \ln x,\ u' = \frac{1}{x},\ v' = x^2,\ v = \frac{1}{3}x^3$	M1	
		$I = [\frac{1}{3}x^3 \ln x]_1^3 - \int_1^3 \frac{1}{3}x^2\ dx$	A1	
		$= [\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3]_1^3$	A1	
		$= (9 \ln 3 - 3) - (0 - \frac{1}{9})$	M1	
		$= 9 \ln 3 - \frac{26}{9}$	A1	**(5)**
4.	*(i)*	$\frac{dx}{dt} = 1 + \cos t, \quad \frac{dy}{dt} = \cos t$	M1	
		$\frac{dy}{dx} = \frac{\cos t}{1+\cos t}$	M1 A1	
	(ii)	$\frac{\cos t}{1+\cos t} = 0,\ \cos t = 0,\ t = \frac{\pi}{2}$	M1 A1	
		$\therefore\ (\frac{\pi}{2} + 1, 1)$	A1	**(6)**
5.		$\int \frac{1}{y^2}\ dy = \int \sqrt{x}\ dx$	M1	
		$-y^{-1} = \frac{2}{3}x^{\frac{3}{2}} + c$	M1 A1	
		$x = 1,\ y = -2 \Rightarrow \frac{1}{2} = \frac{2}{3} + c, \quad c = -\frac{1}{6}$	M1 A1	
		$-\frac{1}{y} = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{6}, \quad \frac{1}{y} = \frac{1}{6} - \frac{2}{3}x^{\frac{3}{2}} = \frac{1}{6}(1 - 4x^{\frac{3}{2}})$	M1	
		$y = \frac{6}{1-4x^{\frac{3}{2}}}$	A1	**(7)**
6.	*(i)*	$= \int (\sec^2 3x - 1)\ dx$	M1	
		$= \frac{1}{3}\tan 3x - x + c$	M1 A1	
	(ii)	$u = x^2 + 4 \Rightarrow \frac{du}{dx} = 2x$	M1	
		$x = 0 \Rightarrow u = 4,\ x = 2 \Rightarrow u = 8$	B1	
		$I = \int_4^8 \frac{5}{2}u^{-2}\ du$	A1	
		$= [-\frac{5}{2}u^{-1}]_4^8$	M1	
		$= -\frac{5}{16} - (-\frac{5}{8}) = \frac{5}{16}$	M1 A1	**(9)**

7.	*(i)*	$6x - 2 + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$	M1 A1	
		$(-1, 3) \Rightarrow -6 - 2 + 3 - \frac{dy}{dx} + 6\frac{dy}{dx} = 0,\quad \frac{dy}{dx} = 1$	M1 A1	
		grad of normal = –1		
		$\therefore\ y - 3 = -(x + 1)$	M1	
		$y = 2 - x$	A1	
	(ii)	sub. $\Rightarrow\ 3x^2 - 2x + x(2 - x) + (2 - x)^2 - 11 = 0$	M1	
		$3x^2 - 4x - 7 = 0$	A1	
		$(3x - 7)(x + 1) = 0$	M1	
		$x = -1$ (at P) or $\frac{7}{3}$ $\therefore (\frac{7}{3}, -\frac{1}{3})$	A1	**(10)**

8.	*(i)*	$\overrightarrow{AB} = (7\mathbf{i} - \mathbf{j} + 12\mathbf{k}) - (-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) = (10\mathbf{i} - 4\mathbf{j} + 10\mathbf{k})$	M1	
		$\therefore\ \mathbf{r} = (-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \lambda(5\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$	A1	
	(ii)	$\overrightarrow{OC} = [\mu\,\mathbf{i} + (5 - 2\mu)\mathbf{j} + (-7 + 7\mu)\mathbf{k}]$		
		$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [(3 + \mu)\mathbf{i} + (2 - 2\mu)\mathbf{j} + (-9 + 7\mu)\mathbf{k}]$	M1 A1	
		$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = [(-7 + \mu)\mathbf{i} + (6 - 2\mu)\mathbf{j} + (-19 + 7\mu)\mathbf{k}]$	A1	
		$\overrightarrow{AC}\,.\,\overrightarrow{BC} = (3 + \mu)(-7 + \mu)+(2 - 2\mu)(6 - 2\mu)+(-9 + 7\mu)(-19 + 7\mu) = 0$	M1	
		$\mu^2 - 4\mu + 3 = 0$	A1	
		$(\mu - 1)(\mu - 3) = 0$	M1	
		$\mu = 1, 3\quad \therefore\ \overrightarrow{OC} = (\mathbf{i} + 3\mathbf{j})$ or $(3\mathbf{i} - \mathbf{j} + 14\mathbf{k})$	A2	
	(iii)	$AC = \sqrt{16+0+4} = 2\sqrt{5},\ BC = \sqrt{36+16+144} = 14$	M1	
		area $= \frac{1}{2} \times 2\sqrt{5} \times 14 = 14\sqrt{5}$	M1 A1	**(13)**

9.	*(i)*	$\frac{8-x}{(1+x)(2-x)} \equiv \frac{A}{1+x} + \frac{B}{2-x}$		
		$8 - x \equiv A(2 - x) + B(1 + x)$	M1	
		$x = -1 \Rightarrow 9 = 3A \Rightarrow A = 3$	A1	
		$x = 2 \Rightarrow 6 = 3B \Rightarrow B = 2\ \therefore\ \mathrm{f}(x) = \frac{3}{1+x} + \frac{2}{2-x}$	A1	
	(ii)	$= \int_0^{\frac{1}{2}} (\frac{3}{1+x} + \frac{2}{2-x})\,dx = [3\ln\lvert 1 + x\rvert - 2\ln\lvert 2 - x\rvert]_0^{\frac{1}{2}}$	M1 A1	
		$= (3\ln\frac{3}{2} - 2\ln\frac{3}{2}) - (0 - 2\ln 2)$	M1	
		$= \ln\frac{3}{2} + \ln 4 = \ln 6$	A1	
	(iii)	$\mathrm{f}(x) = 3(1 + x)^{-1} + 2(2 - x)^{-1}$		
		$(1 + x)^{-1} = 1 - x + x^2 - x^3 + \ldots$	B1	
		$(2 - x)^{-1} = 2^{-1}(1 - \frac{1}{2}x)^{-1}$	M1	
		$= \frac{1}{2}[1 + (-1)(-\frac{1}{2}x) + \frac{(-1)(-2)}{2}(-\frac{1}{2}x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-\frac{1}{2}x)^3 + \ldots]$	M1	
		$= \frac{1}{2}(1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots)$	A1	
		$\therefore\ \mathrm{f}(x) = 3(1 - x + x^2 - x^3 + \ldots) + (1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots)$	M1	
		$= 4 - \frac{5}{2}x + \frac{13}{4}x^2 - \frac{23}{8}x^3 + \ldots$	A1	**(13)**

Total **(72)**