Core Mathematics 4 Paper C

1. A curve has the equation

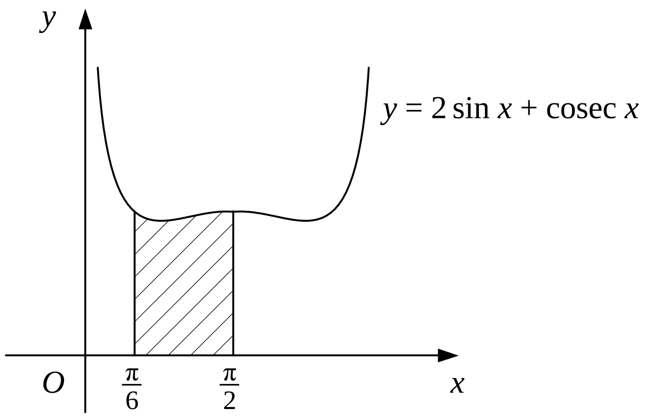
$$x^2(2 + y) - y^2 = 0.$$

Find an expression for $\frac{dy}{dx}$ in terms of x and y. [5]

2. Show that

$$\int_1^2 x \ln x \, dx = 2 \ln 2 - \tfrac{3}{4}.$$ [5]

3.



The diagram shows the curve with equation $y = 2 \sin x + \operatorname{cosec} x$, $0 < x < \pi$.

The shaded region bounded by the curve, the x-axis and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$ is rotated through four right angles about the x-axis.

Show that the volume of the solid formed is $\frac{1}{2}\pi(4\pi + 3\sqrt{3})$. [7]

4. *(i)* Express

$$\frac{4x}{x^2 - 9} - \frac{2}{x + 3}$$

as a single fraction in its simplest form. [3]

(ii) Simplify

$$\frac{x^3 - 8}{3x^2 - 8x + 4}.$$ [5]

5. A bath is filled with hot water which is allowed to cool. The temperature of the water is θ°C after cooling for t minutes and the temperature of the room is assumed to remain constant at 20°C.

Given that the rate at which the temperature of the water decreases is proportional to the difference in temperature between the water and the room,

(i) write down a differential equation connecting θ and t. [1]

Given also that the temperature of the water is initially 37°C and that it is 36°C after cooling for four minutes,

(ii) find, to 3 significant figures, the temperature of the water after ten minutes. [8]

Advice suggests that the temperature of the water should be allowed to cool to 33°C before a child gets in.

(iii) Find, to the nearest second, how long a child should wait before getting into the bath. [2]

6. A curve has parametric equations

$$x = 3\cos^2 t, \quad y = \sin 2t, \quad 0 \le t < \pi.$$

(i) Show that $\dfrac{dy}{dx} = -\tfrac{2}{3}\cot 2t$. [3]

(ii) Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. [3]

(iii) Show that the tangent to the curve at the point where $t = \frac{\pi}{6}$ has the equation

$$2x + 3\sqrt{3}\,y = 9.$$ [3]

(iv) Find a cartesian equation for the curve in the form $y^2 = f(x)$. [3]

Turn over

7. Relative to a fixed origin, the points A and B have position vectors $\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix}$ respectively.

(i) Find a vector equation for the line l_1 which passes through A and B. [2]

The line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ -7 \\ 9 \end{pmatrix} + t\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}.$$

(ii) Show that lines l_1 and l_2 do not intersect. [4]

(iii) Find the position vector of the point C on l_2 such that $\angle ABC = 90°$. [6]

8.

$$\mathrm{f}(x) = \frac{5-8x}{(1+2x)(1-x)^2}.$$

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Find the series expansion of $\mathrm{f}(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [6]

(iii) State the set of values of x for which your expansion is valid. [1]