C4 Paper C – Marking Guide

1. $2x(2+y) + x^2\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$ — M2 A1

$\frac{dy}{dx} = \frac{2x(2+y)}{2y-x^2}$ — M1 A1 **(5)**

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2. $u = \ln x,\ u' = \frac{1}{x},\ v' = x,\ v = \frac{1}{2}x^2$ — M1

$I = [\frac{1}{2}x^2 \ln x]_1^2 - \int_1^2 \frac{1}{2}x\ dx$ — A1

$= [\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2]_1^2$ — M1

$= (2\ln 2 - 1) - (0 - \frac{1}{4}) = 2\ln 2 - \frac{3}{4}$ — M1 A1 **(5)**

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3. $= \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2\sin x + \operatorname{cosec} x)^2\ dx = \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4\sin^2 x + 4 + \operatorname{cosec}^2 x)\ dx$ — M1 A1

$= \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 - 2\cos 2x + 4 + \operatorname{cosec}^2 x)\ dx$ — M1

$= \pi[6x - \sin 2x - \cot x]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ — M1 A1

$= \pi\{(3\pi + 0 + 0) - (\pi - \frac{\sqrt{3}}{2} - \sqrt{3})\}$ — M1

$= \pi(2\pi + \frac{3}{2}\sqrt{3}) = \frac{1}{2}\pi(4\pi + 3\sqrt{3})$ — A1 **(7)**

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4. *(i)* $= \frac{4x}{(x+3)(x-3)} - \frac{2}{x+3} = \frac{4x-2(x-3)}{(x+3)(x-3)}$ — M1

$= \frac{2x+6}{(x+3)(x-3)} = \frac{2(x+3)}{(x+3)(x-3)} = \frac{2}{x-3}$ — M1 A1

(ii) $2^3 - 8 = 0\ \therefore (x-2)$ is a factor of $(x^3 - 8)$ — B1

$$\begin{array}{r|l} x-2 & x^2 + 2x + 4 \\ \hline & x^3 + 0x^2 + 0x - 8 \\ & x^3 - 2x^2 \\ \hline & \quad 2x^2 + 0x \\ & \quad 2x^2 - 4x \\ \hline & \quad\quad 4x - 8 \\ & \quad\quad 4x - 8 \end{array}$$

— M1 A1

$\therefore\ x^3 - 8 = (x-2)(x^2 + 2x + 4)$

$\therefore\ \frac{x^3-8}{3x^2-8x+4} = \frac{(x-2)(x^2+2x+4)}{(3x-2)(x-2)} = \frac{x^2+2x+4}{3x-2}$ — M1 A1 **(8)**

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5. *(i)* $\frac{d\theta}{dt} = -k(\theta - 20)$ — B1

(ii) $\int \frac{1}{\theta-20}\ d\theta = \int -k\ dt$ — M1

$\ln|\theta - 20| = -kt + c$ — M1 A1

$t = 0,\ \theta = 37 \Rightarrow c = \ln 17$ — M1

$\ln|\frac{\theta-20}{17}| = -kt,\quad \theta = 20 + 17e^{-kt}$ — A1

$t = 4,\ \theta = 36 \Rightarrow\quad 36 = 20 + 17e^{-4k}$ — M1

$k = -\frac{1}{4}\ln\frac{16}{17} = 0.01516$ — A1

$t = 10,\ \theta = 20 + 17e^{-0.01516\times 10} = 34.6°C$ (3sf) — A1

(iii) $33 = 20 + 17e^{-0.01516t}$

$t = -\frac{1}{0.01516}\ln\frac{13}{17} = 17.70$ minutes $= 17$ mins 42 secs — M1 A1 **(11)**

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6. *(i)* $\dfrac{dx}{dt} = 6\cos t \times (-\sin t), \quad \dfrac{dy}{dt} = 2\cos 2t$ M1

$\dfrac{dy}{dx} = \dfrac{2\cos 2t}{-6\cos t \sin t} = \dfrac{2\cos 2t}{-3\sin 2t} = -\frac{2}{3}\cot 2t$ M1 A1

(ii) $-\frac{2}{3}\cot 2t = 0 \Rightarrow 2t = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow t = \frac{\pi}{4}, \frac{3\pi}{4}$ M1 A1

$\therefore\ (\frac{3}{2}, 1),\ (\frac{3}{2}, -1)$ A1

(iii) $t = \frac{\pi}{6},\ x = \frac{9}{4},\ y = \frac{\sqrt{3}}{2},$ grad $= -\frac{2}{3\sqrt{3}}$ B1

$\therefore\ y - \frac{\sqrt{3}}{2} = -\frac{2}{3\sqrt{3}}(x - \frac{9}{4})$ M1

$6\sqrt{3}\,y - 9 = -4x + 9$

$2x + 3\sqrt{3}\,y = 9$ A1

(iv) $y^2 = \sin^2 2t = 4\sin^2 t \cos^2 t = 4(1 - \cos^2 t)\cos^2 t$ M1

$\cos^2 t = \frac{x}{3} \quad \therefore y^2 = 4(1 - \frac{x}{3})\frac{x}{3},\ \ y^2 = \frac{4}{9}x(3 - x)$ M1 A1 **(12)**

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7. *(i)* $\overrightarrow{AB} = \begin{pmatrix}-3\\6\\1\end{pmatrix} - \begin{pmatrix}-4\\1\\3\end{pmatrix} = \begin{pmatrix}1\\5\\-2\end{pmatrix} \quad \therefore\ \mathbf{r} = \begin{pmatrix}-4\\1\\3\end{pmatrix} + s\begin{pmatrix}1\\5\\-2\end{pmatrix}$ M1 A1

(ii) $-4 + s = 3 + 2t \quad (1)$

$1 + 5s = -7 - 3t \quad (2)$

$3 - 2s = 9 + t \quad (3)$ B1

$2 \times (1) + (3)$: $-5 = 15 + 5t,\ t = -4,\ s = -1$ M1 A1

sub. (2): $1 - 5 = -7 + 12$, not true $\therefore$ do not intersect A1

(iii) $\overrightarrow{OC} = \begin{pmatrix}3+2t\\-7-3t\\9+t\end{pmatrix},\ \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix}6+2t\\-13-3t\\8+t\end{pmatrix}$ M1 A1

$\therefore\ \begin{pmatrix}1\\5\\-2\end{pmatrix} . \begin{pmatrix}6+2t\\-13-3t\\8+t\end{pmatrix} = 0, \quad 6 + 2t - 65 - 15t - 16 - 2t = 0$ M1 A1

$t = -5 \ \therefore\ \overrightarrow{OC} = \begin{pmatrix}-7\\8\\4\end{pmatrix}$ M1 A1 **(12)**

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8. *(i)* $\dfrac{5-8x}{(1+2x)(1-x)^2} \equiv \dfrac{A}{1+2x} + \dfrac{B}{1-x} + \dfrac{C}{(1-x)^2}$

$5 - 8x \equiv A(1 - x)^2 + B(1 + 2x)(1 - x) + C(1 + 2x)(1 - x)$ M1

$x = -\frac{1}{2} \Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4$ A1

$x = 1 \Rightarrow -3 = 3C \Rightarrow C = -1$ A1

coeffs $x^2 \Rightarrow 0 = A - 2B \Rightarrow B = 2$ M1 A1

$f(x) = \dfrac{4}{1+2x} + \dfrac{2}{1-x} - \dfrac{1}{(1-x)^2}$

(ii) $f(x) = 4(1 + 2x)^{-1} + 2(1 - x)^{-1} - (1 - x)^{-2}$

$(1 + 2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(2x)^3 + \ldots$ M1

$= 1 - 2x + 4x^2 - 8x^3 + \ldots$ A1

$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots$ B1

$(1 - x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \frac{(-2)(-3)(-4)}{3\times 2}(-x)^3 + \ldots$

$= 1 + 2x + 3x^2 + 4x^3 + \ldots$ A1

$f(x) = 4(1 - 2x + 4x^2 - 8x^3) + 2(1 + x + x^2 + x^3) - (1 + 2x + 3x^2 + 4x^3)$ M1

$= 5 - 8x + 15x^2 - 34x^3 + \ldots$ A1

(iii) $|x| < \frac{1}{2}$ A1 **(12)**

Total **(72)**