

# Core Mathematics 4 Paper B

1. Find  $\int x e^{3x} dx$ . [4]

2. Find the quotient and remainder when  $(x^4 + x^3 - 5x^2 - 9)$  is divided by  $(x^2 + x - 6)$ . [4]

3. Differentiate each of the following with respect to  $x$  and simplify your answers.

(i)  $\cot x^2$  [2]

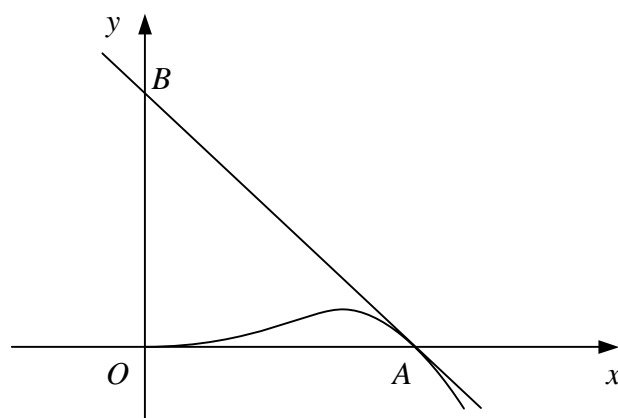
(ii)  $\frac{\sin x}{3 + 2 \cos x}$  [4]

4. (i) Expand  $(1 - 3x)^{-2}$ ,  $|x| < \frac{1}{3}$ , in ascending powers of  $x$  up to and including the term in  $x^3$ , simplifying each coefficient. [4]

(ii) Hence, or otherwise, show that for small  $x$ ,

$$\left(\frac{2-x}{1-3x}\right)^2 \approx 4 + 20x + 85x^2 + 330x^3. \quad [3]$$

5.



The diagram shows the curve with parametric equations

$$x = a\sqrt{t}, \quad y = at(1-t), \quad t \geq 0,$$

where  $a$  is a positive constant.

(i) Find  $\frac{dy}{dx}$  in terms of  $t$ . [3]

The curve meets the  $x$ -axis at the origin,  $O$ , and at the point  $A$ . The tangent to the curve at  $A$  meets the  $y$ -axis at the point  $B$  as shown.

(ii) Show that the area of triangle  $OAB$  is  $a^2$ . [5]

6. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = (7\mathbf{j} - 4\mathbf{k}) + s(4\mathbf{i} - 3\mathbf{j} + \mathbf{k}),$$

and

$$\mathbf{r} = (-7\mathbf{i} + \mathbf{j} + 8\mathbf{k}) + t(-3\mathbf{i} + 2\mathbf{k}),$$

where  $s$  and  $t$  are scalar parameters.

- (i) Show that the two lines intersect and find the position vector of the point where they meet. [5]
- (ii) Find, in degrees to 1 decimal place, the acute angle between the lines. [4]

7. At time  $t = 0$ , a tank of height 2 metres is completely filled with water. Water then leaks from a hole in the side of the tank such that the depth of water in the tank,  $y$  metres, after  $t$  hours satisfies the differential equation

$$\frac{dy}{dt} = -ke^{-0.2t},$$

where  $k$  is a positive constant,

- (i) Find an expression for  $y$  in terms of  $k$  and  $t$ . [4]

Given that two hours after being filled the depth of water in the tank is 1.6 metres,

- (ii) find the value of  $k$  to 4 significant figures. [2]

Given also that the hole in the tank is  $h$  cm above the base of the tank,

- (iii) show that  $h = 79$  to 2 significant figures. [3]

**Turn over**

8. A curve has the equation

$$x^2 - 4xy + 2y^2 = 1.$$

- (i) Find an expression for  $\frac{dy}{dx}$  in its simplest form in terms of  $x$  and  $y$ . [4]

- (ii) Show that the tangent to the curve at the point  $P(1, 2)$  has the equation

$$3x - 2y + 1 = 0. \quad [3]$$

The tangent to the curve at the point  $Q$  is parallel to the tangent at  $P$ .

- (iii) Find the coordinates of  $Q$ . [4]

9. (i) Show that the substitution  $u = \sin x$  transforms the integral

$$\int \frac{6}{\cos x(2 - \sin x)} dx$$

into the integral

$$\int \frac{6}{(1 - u^2)(2 - u)} du. \quad [4]$$

- (ii) Express  $\frac{6}{(1 - u^2)(2 - u)}$  in partial fractions. [4]

- (iii) Hence, evaluate

$$\int_0^{\frac{\pi}{6}} \frac{6}{\cos x(2 - \sin x)} dx,$$

giving your answer in the form  $a \ln 2 + b \ln 3$ , where  $a$  and  $b$  are integers. [6]