C4 Paper B – Marking Guide

1.		$u = x,\ u' = 1,\ v' = e^{3x},\ v = \frac{1}{3}e^{3x}$	M1 A1	
		$I = \frac{1}{3}x\,e^{3x} - \int \frac{1}{3}e^{3x}\,dx$	M1	
		$= \frac{1}{3}x\,e^{3x} - \frac{1}{9}e^{3x} + c$	A1	**(4)**
2.		$$\begin{array}{r|l} & x^2 + 0x + 1 \\ x^2+x-6 & x^4 + x^3 - 5x^2 + 0x - 9 \\ & x^4 + x^3 - 6x^2 \\ & \quad x^2 + 0x - 9 \\ & \quad x^2 + x - 6 \\ & \quad\quad -x - 3 \end{array}$$	M2	
		$\therefore$ quotient $= x^2 + 1$, remainder $= -x - 3$	A2	**(4)**
3.	*(i)*	$= -\text{cosec}^2 x^2 \times 2x = -2x\,\text{cosec}^2 x^2$	M1 A1	
	(ii)	$= \dfrac{\cos x\times(3+2\cos x)-\sin x\times(-2\sin x)}{(3+2\cos x)^2}$	M1 A1	
		$= \dfrac{3\cos x+2\cos^2 x+2\sin^2 x}{(3+2\cos x)^2} = \dfrac{3\cos x+2}{(3+2\cos x)^2}$	M1 A1	**(6)**
4.	*(i)*	$(1-3x)^{-2} = 1 + (-2)(-3x) + \frac{(-2)(-3)}{2}(-3x)^2 + \frac{(-2)(-3)(-4)}{3\times 2}(-3x)^3 + \ldots$	M1	
		$= 1 + 6x + 27x^2 + 108x^3 + \ldots$	A3	
	(ii)	$\left(\frac{2-x}{1-3x}\right)^2 = (2-x)^2(1-3x)^{-2} = (4-4x+x^2)(1+6x+27x^2+108x^3+\ldots)$	M1	
		$= 4 + 24x + 108x^2 + 432x^3 - 4x - 24x^2 - 108x^3 + x^2 + 6x^3 + \ldots$	A1	
		$\therefore$ for small x, $\left(\frac{2-x}{1-3x}\right)^2 = 4 + 20x + 85x^2 + 330x^3$	A1	**(7)**
5.	*(i)*	$\frac{dx}{dt} = \frac{1}{2}at^{-\frac{1}{2}}$, $\quad \frac{dy}{dt} = a(1-2t)$	M1	
		$\frac{dy}{dx} = \dfrac{a(1-2t)}{\frac{1}{2}at^{-\frac{1}{2}}} = 2\sqrt{t}\,(1-2t)$	M1 A1	
	(ii)	$y = 0 \Rightarrow t = 0$ (at O) or 1 (at A)	B1	
		$t = 1,\ x = a,\ y = 0$, grad $= -2$	M1	
		$\therefore\ y - 0 = -2(x - a)$	A1	
		at B, $x = 0\ \therefore\ y = 2a$	M1	
		area $= \frac{1}{2} \times a \times 2a = a^2$	A1	**(8)**
6.	*(i)*	$4s = -7 - 3t$ (1)		
		$7 - 3s = 1$ (2)		
		$-4 + s = 8 + 2t$ (3)	M1	
		(2) $\Rightarrow s = 2$, sub. (1) $\Rightarrow t = -5$	B1 M1	
		check (3) $-4 + 2 = 8 - 10$, true $\therefore$ intersect	A1	
		intersect at $(7\mathbf{j} - 4\mathbf{k}) + 2(4\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = (8\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	A1	
	(ii)	$= \cos^{-1}\left\lvert\dfrac{4\times(-3)+(-3)\times 0+1\times 2}{\sqrt{16+9+1}\times\sqrt{9+0+4}}\right\rvert$	M1 A1	
		$= \cos^{-1}\left\lvert\dfrac{-10}{\sqrt{26}\times\sqrt{13}}\right\rvert = 57.0°$ (1dp)	M1 A1	**(9)**

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7.	*(i)*	$\int \mathrm{d}y = \int -k\mathrm{e}^{-0.2t}\ \mathrm{d}t$	M1	
		$y = 5k\mathrm{e}^{-0.2t} + c$	A1	
		$t = 0,\ y = 2 \Rightarrow 2 = 5k + c, \quad c = 2 - 5k$	M1	
		$\therefore\ y = 5k\mathrm{e}^{-0.2t} - 5k + 2$	A1	
	(ii)	$t = 2,\ y = 1.6 \Rightarrow 1.6 = 5k\mathrm{e}^{-0.4} - 5k + 2$		
		$k = \dfrac{-0.4}{5\mathrm{e}^{-0.4} - 5} = 0.2427$ (4sf)	M1 A1	
	(iii)	as $t \to \infty,\ y \to h$ (in metres)	M1	
		$\therefore$ "h" $= -5k + 2 = 0.787$ m $= 78.7$ cm $\therefore\ h = 79$ (2sf)	M1 A1	**(9)**

8.	*(i)*	$2x - 4y - 4x\dfrac{\mathrm{d}y}{\mathrm{d}x} + 4y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1	
		$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x-4y}{4x-4y} = \dfrac{x-2y}{2x-2y}$	M1 A1	
	(ii)	grad $= \frac{3}{2}$	M1	
		$\therefore\ y - 2 = \frac{3}{2}(x - 1)$	M1	
		$2y - 4 = 3x - 3$		
		$3x - 2y + 1 = 0$	A1	
	(iii)	$\dfrac{x-2y}{2x-2y} = \dfrac{3}{2}$	M1	
		$2(x - 2y) = 3(2x - 2y), \quad y = 2x$	A1	
		sub. $\Rightarrow\ x^2 - 8x^2 + 8x^2 = 1$	M1	
		$x^2 = 1,\ \ x = 1$ (at P) or -1		
		$\therefore\ Q\,(-1, -2)$	A1	**(11)**

9.	*(i)*	$u = \sin x \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = \cos x$	B1	
		$I = \displaystyle\int \frac{6\cos x}{\cos^2 x(2-\sin x)}\,\mathrm{d}x = \int \frac{6\cos x}{(1-\sin^2 x)(2-\sin x)}\,\mathrm{d}x$	M1	
		$= \displaystyle\int \frac{6}{(1-u^2)(2-u)}\,\mathrm{d}u$	M1 A1	
	(ii)	$\dfrac{6}{(1+u)(1-u)(2-u)} \equiv \dfrac{A}{1+u} + \dfrac{B}{1-u} + \dfrac{C}{2-u}$		
		$6 \equiv A(1 - u)(2 - u) + B(1 + u)(2 - u) + C(1 + u)(1 - u)$	M1	
		$u = -1 \Rightarrow 6 = 6A \Rightarrow A = 1$	A1	
		$u = 1 \Rightarrow 6 = 2B \Rightarrow B = 3$	A1	
		$u = 2 \Rightarrow 6 = -3C \Rightarrow C = -2$	A1	
		$\therefore\ \dfrac{6}{(1-u^2)(2-u)} \equiv \dfrac{1}{1+u} + \dfrac{3}{1-u} - \dfrac{2}{2-u}$		
	(iii)	$x = 0 \Rightarrow u = 0,\ x = \frac{\pi}{6} \Rightarrow u = \frac{1}{2}$	M1	
		$I = \displaystyle\int_0^{\frac{1}{2}} \left(\frac{1}{1+u} + \frac{3}{1-u} - \frac{2}{2-u}\right)\mathrm{d}u$		
		$= [\ln\lvert 1 + u\rvert - 3\ln\lvert 1 - u\rvert + 2\ln\lvert 2 - u\rvert]_0^{\frac{1}{2}}$	M1 A1	
		$= (\ln\frac{3}{2} - 3\ln\frac{1}{2} + 2\ln\frac{3}{2}) - (0 + 0 + 2\ln 2)$	M1	
		$= 3\ln\frac{3}{2} + 3\ln 2 - 2\ln 2$		
		$= 3\ln 3 - 3\ln 2 + \ln 2 = 3\ln 3 - 2\ln 2$	M1 A1	**(14)**

Total **(72)**

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