Core Mathematics 4 Paper A

1. Express

$$\frac{2x}{2x^2+3x-5} \div \frac{x^3}{x^2-x}$$

as a single fraction in its simplest form. [4]

2. A curve has the equation

$$2x^2 + xy - y^2 + 18 = 0.$$

Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. [7]

3. The first four terms in the series expansion of $(1 + ax)^n$ in ascending powers of x are

$$1 - 4x + 24x^2 + kx^3,$$

where a, n and k are constants and $|ax| < 1$.

(i) Find the values of a and n. [6]

(ii) Show that $k = -160$. [2]

4. Relative to a fixed origin, O, the points A and B have position vectors $\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$ respectively.

Find, in exact, simplified form,

(i) the cosine of $\angle AOB$, [4]

(ii) the area of triangle OAB, [3]

(iii) the shortest distance from A to the line OB. [2]

5. *(i)* Use the derivatives of $\sin x$ and $\cos x$ to prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x.$$ [4]

The tangent to the curve $y = 2x \tan x$ at the point where $x = \frac{\pi}{4}$ meets the y-axis at the point P.

(ii) Find the y-coordinate of P in the form $k\pi^2$ where k is a rational constant. [6]

6. *(i)* Find

$$\int \cot^2 2x \; \mathrm{d}x.$$ [3]

(ii) Use the substitution $u^2 = x + 1$ to evaluate

$$\int_0^3 \frac{x^2}{\sqrt{x+1}} \; \mathrm{d}x.$$ [7]

7. During a chemical reaction, a compound is being made from two other substances.

At time t hours after the start of the reaction, x g of the compound has been produced. Assuming that $x = 0$ initially, and that

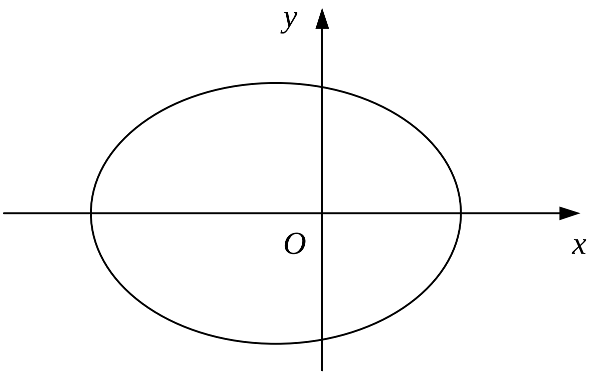
$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2(x - 6)(x - 3),$$

(i) show that it takes approximately 7 minutes to produce 2 g of the compound. [10]

(ii) Explain why it is not possible to produce 3 g of the compound. [2]

Turn over

8.



The diagram shows the curve with parametric equations

$$x = -1 + 4\cos\theta, \quad y = 2\sqrt{2}\sin\theta, \quad 0 \leq \theta < 2\pi.$$

The point P on the curve has coordinates $(1, \sqrt{6})$.

(i) Find the value of θ at P. [2]

(ii) Show that the normal to the curve at P passes through the origin. [7]

(iii) Find a cartesian equation for the curve. [3]