OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4724

Core Mathematics 4

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

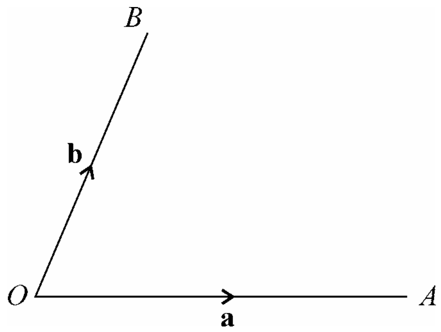
1 Find the quotient and remainder when x^4+1 is divided by x^2+1. [4]

2 **(i)** Expand $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3. [4]

(ii) State the set of values for which the expansion in part **(i)** is valid. [1]

3 Find $\int_0^1 x\mathrm{e}^{-2x}\,\mathrm{d}x$, giving your answer in terms of e. [5]

4



As shown in the diagram the points A and B have position vectors **a** and **b** with respect to the origin O.

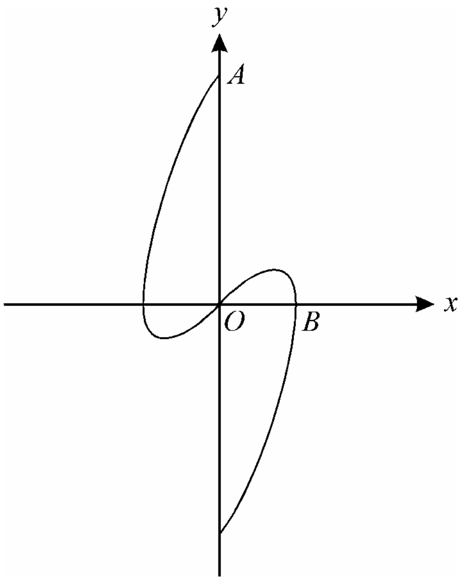
(i) Make a sketch of the diagram, and mark the points C, D and E such that $\overrightarrow{OC}=2\mathbf{a}$, $\overrightarrow{OD}=2\mathbf{a}+\mathbf{b}$ and $\overrightarrow{OE}=\frac{1}{3}\overrightarrow{OD}$. [3]

(ii) By expressing suitable vectors in terms of **a** and **b**, prove that E lies on the line joining A and B. [4]

5 **(i)** For the curve $2x^2+xy+y^2=14$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(ii) Deduce that there are two points on the curve $2x^2+xy+y^2=14$ at which the tangents are parallel to the x-axis, and find their coordinates. [4]

6



The diagram shows the curve with parametric equations

$$x = a\sin\theta, \quad y = a\theta\cos\theta,$$

where a is a positive constant and $-\pi \leqslant \theta \leqslant \pi$. The curve meets the positive y-axis at A and the positive x-axis at B.

(i) Write down the value of θ corresponding to the origin, and state the coordinates of A and B. [3]

(ii) Show that $\dfrac{dy}{dx} = 1 - \theta\tan\theta$, and hence find the equation of the tangent to the curve at the origin. [6]

7 The line L_1 passes through the point $(3, 6, 1)$ and is parallel to the vector $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. The line L_2 passes through the point $(3, -1, 4)$ and is parallel to the vector $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

(i) Write down vector equations for the lines L_1 and L_2. [2]

(ii) Prove that L_1 and L_2 intersect, and find the coordinates of their point of intersection. [5]

(iii) Calculate the acute angle between the lines. [4]

8 Let $I = \displaystyle\int \frac{1}{x(1+\sqrt{x})^2}\,dx$.

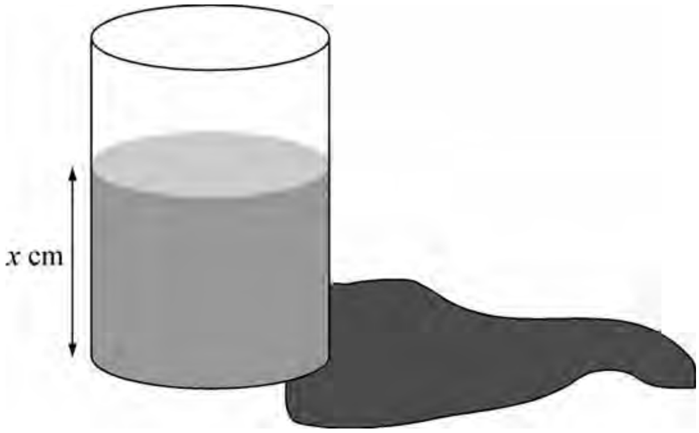
(i) Show that the substitution $u = \sqrt{x}$ transforms I to $\displaystyle\int \frac{2}{u(1+u)^2}\,du$. [3]

(ii) Express $\dfrac{2}{u(1+u)^2}$ in the form $\dfrac{A}{u} + \dfrac{B}{1+u} + \dfrac{C}{(1+u)^2}$. [5]

(ii) Hence find I. [4]

[Turn over

9



A cylindrical container has a height of 200 cm. The container was initially full of a chemical but there is a leak from a hole in the base. When the leak is noticed, the container is half-full and the level of the chemical is dropping at a rate of 1 cm per minute. It is required to find for how many minutes the container has been leaking. To model the situation it is assumed that, when the depth of the chemical remaining is x cm, the rate at which the level is dropping is proportional to $\sqrt{x}$.

Set up and solve an appropriate differential equation, and hence show that the container has been leaking for about 80 minutes. [11]