

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4724

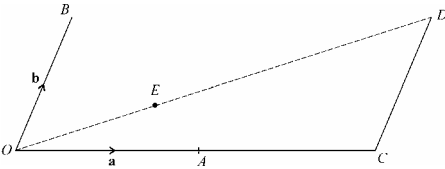
Core Mathematics 4

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72
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This mark scheme consists of 4 printed pages.

<p>1 $\frac{x^4 + 1}{x^2 + 1} = x^2 - 1 + \frac{2}{x^2 + 1}$</p>	<p>B1 M1 A1 A1</p>	<p>For correct leading term x^2 in quotient For evidence of correct division process For correct quotient $x^2 - 1$ For correct remainder 2</p> <p style="text-align: right;">4 4</p>
<p>2 (i) $(1 - 2x)^{-\frac{1}{2}} = 1 + (-\frac{1}{2})(-2x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-2x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-2x)^3 + \dots$ $= 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3$</p>	<p>M1 A1 A1 A1</p>	<p>For 2nd, 3rd or 4th term OK (unsimplified) For $1 + x$ correct For $+\frac{3}{2}x^2$ correct For $+\frac{5}{2}x^3$ correct</p>
<p>(ii) Valid for $x < \frac{1}{2}$</p>	<p>B1</p>	<p>For any correct expression(s)</p> <p style="text-align: right;">1 5</p>
<p>3 $\int_0^1 x e^{-2x} dx = \left[-\frac{1}{2} x e^{-2x} \right]_0^1 - \int_0^1 -\frac{1}{2} e^{-2x} dx$ $= \left[-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} \right]_0^1$ $= \frac{1}{4} - \frac{3}{4} e^{-2}$</p>	<p>M1 A1 M1 M1 A1</p>	<p>For attempt at 'parts' going the correct way For correct terms $-\frac{1}{2} x e^{-2x} - \int -\frac{1}{2} e^{-2x} dx$ For consistent attempt at second integration For correct use of limits throughout For correct (exact) answer in any form</p> <p style="text-align: right;">5 5</p>
<p>4 (i) </p>	<p>B1 B1 B1✓</p>	<p>For C correctly located on sketch For D correctly located on sketch For E correctly located wrt O and D</p> <p style="text-align: right;">3</p>
<p>(ii) $\overline{AE} = \frac{1}{3}(2\mathbf{a} + \mathbf{b}) - \mathbf{a} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$ Hence AE is parallel to AB i.e. E lies on the line joining A to B</p>	<p>M1 A1 A1 A1</p>	<p>For relevant subtraction involving \overline{OE} For correct expression for $(\pm)\overline{AE}$ or \overline{EB} For correct recognition of parallel property For complete proof of required result</p> <p style="text-align: right;">4 7</p>
<p>5 (i) $4x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$ Hence $\frac{dy}{dx} = -\frac{4x + y}{x + 2y}$</p>	<p>B1 B1 M1 A1</p>	<p>For correct terms $x \frac{dy}{dx} + y$ For correct term $2y \frac{dy}{dx}$ For solving for $\frac{dy}{dx}$ For any correct form of expression</p>
<p>(ii) $\frac{dy}{dx} = 0 \Rightarrow y = -4x$ Hence $2x^2 + (-4x)^2 + (-4x)^2 = 14$ i.e. $x^2 = 1$ So the two points are (1, -4) and (-1, 4)</p>	<p>M1 M1 A1 A1</p>	<p>For stating or using their $\frac{dy}{dx} = 0$ For solving simultaneously with curve equ For correct value of x^2 (or y^2) For both correct points identified</p> <p style="text-align: right;">4 8</p>

<p>6 (i) $\theta = 0$ at the origin A is $(0, a\pi)$ B is $(a, 0)$</p> <hr/> <p>(ii) $\frac{dx}{d\theta} = a \cos \theta$ $\frac{dy}{d\theta} = a(\cos \theta - \theta \sin \theta)$ Hence $\frac{dy}{dx} = \frac{\cos \theta - \theta \sin \theta}{\cos \theta} = 1 - \theta \tan \theta$ Gradient of tangent at the origin is 1 Hence equation is $y = x$</p>	<p>B1 B1 B1 B1 M1 M1 A1 M1 A1</p>	<p>For the correct value For the correct y-coordinate at A For the correct x-coordinate at B</p> <hr/> <p>For correct differentiation of x For differentiating y using product rule For use of $\frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta}$ For given result correctly obtained For using $\theta = 0$ For correct equation</p> <p style="text-align: right;">3</p> <hr/> <p style="text-align: right;">6</p> <p style="text-align: right;">9</p>
<p>6 (i) $L_1 : \mathbf{r} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ $L_2 : \mathbf{r} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$</p> <hr/> <p>(ii) $3 + 2s = 3 + t, 6 + 3s = -1 - 2t, 1 - s = 4 + t$ First pair of equations give $s = -1, t = -2$ Third equation checks: $1 + 1 = 4 - 2$ Point of intersection is $(1, 3, 2)$</p> <hr/> <p>(iii) $2 \times 1 + 3 \times (-2) + (-1) \times 1 = (\sqrt{14})(\sqrt{6}) \cos \theta$ Hence acute angle is 56.9°</p>	<p>M1 A1 M1 M1 A1 A1 A1 B1 B1 M1 A1</p>	<p>For correct RHS structure for either line For both lines correct</p> <hr/> <p>For at least 2 equations with two parameters For solving any relevant pair of equations For both parameters correct For explicit check in unused equation For correct coordinates</p> <hr/> <p>For scalar product of correct direction vectors For correct magnitudes $\sqrt{14}$ and $\sqrt{6}$ For correct process for $\cos \theta$ with any pair of vectors relevant to these lines For correct acute angle</p> <p style="text-align: right;">2</p> <hr/> <p style="text-align: right;">5</p> <hr/> <p style="text-align: right;">4</p> <p style="text-align: right;">11</p>
<p>8 (i) $I = \int \frac{1}{u^2(1+u)^2} \times 2u \, du = \int \frac{2}{u(1+u)^2} \, du$</p> <hr/> <p>(ii) $2 \equiv A(1+u)^2 + Bu(1+u) + Cu$ $A = 2$ $C = -2$ $0 = A + B$ (e.g.) $B = -2$</p> <hr/> <p>(iii) $2 \ln u - 2 \ln(1+u) + \frac{2}{1+u}$ Hence $I = \ln x - 2 \ln(1 + \sqrt{x}) + \frac{2}{1 + \sqrt{x}} + c$</p>	<p>M1 A1 A1 M1 B1 B1 A1 A1 B1✓ B1✓ M1 A1</p>	<p>For any attempt to find $\frac{dx}{du}$ or $\frac{du}{dx}$ For '$dx = 2u \, du$' or equivalent correctly used For showing the given result correctly</p> <hr/> <p>For correct identity stated For correct value stated For correct value stated For any correct equation involving B For correct value</p> <hr/> <p>For $A \ln u + B \ln(1+u)$ with their values For $-C(1+u)^{-1}$ with their value For substituting back For completely correct answer (excluding c)</p> <p style="text-align: right;">3</p> <hr/> <p style="text-align: right;">5</p> <hr/> <p style="text-align: right;">4</p> <p style="text-align: right;">12</p>

<p>9 $\frac{dx}{dt} = -k\sqrt{x}$</p> <p>$x = 100$ and $\frac{dx}{dt} = -1 \Rightarrow k = 0.1$</p> <p>Hence equation is $\frac{dx}{dt} = -0.1\sqrt{x}$</p> <p>$\int x^{-\frac{1}{2}} dx = -0.1 \int dt \Rightarrow 2x^{\frac{1}{2}} = -0.1t + c$</p> <p>$x = 200, t = 0 \Rightarrow c = 2\sqrt{200}$</p> <p>So when $x = 100, 2\sqrt{100} = -0.1t + 2\sqrt{200}$</p> <p>i.e. $t = 82.8$</p>	M1	For use of derivative for rate of change
	A1	For correct equation (neg sign optional here)
	M1	For use of data and their DE to find k
	A1	For any form of correct DE
	M1	For separation and integration of both sides
	A1	For $2x^{\frac{1}{2}}$ correct
	A1✓	For $(\pm)kt$ correct (the numerical evaluation of k may be delayed until after the DE is solved)
	B1	For one arbitrary constant included (or equivalent statement of both pairs of limits)
	M1	For evaluation of c
	M1	For evaluation of t
	A1	11 For correct value 82.8 (minutes)
	11	