**OCR**
RECOGNISING ACHIEVEMENT

**ADVANCED GCE**

**MATHEMATICS** **4724**

Core Mathematics 4

**QUESTION PAPER**

Candidates answer on the printed answer book.

**OCR supplied materials:**
*   Printed answer book 4724
*   List of Formulae (MF1)

**Other materials required:**
*   Scientific or graphical calculator

**Thursday 16 June 2011**
**Afternoon**

**Duration:** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the printed answer book and the question paper.

*   The question paper will be found in the centre of the printed answer book.
*   Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
*   **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
*   Use black ink. Pencil may be used for graphs and diagrams only.
*   Read each question carefully. Make sure you know what you have to do before starting your answer.
*   Answer **all** the questions.
*   Do **not** write in the bar codes.
*   You are permitted to use a scientific or graphical calculator in this paper.
*   Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the printed answer book and the question paper.

*   The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
*   **You are reminded of the need for clear presentation in your answers.**
*   The total number of marks for this paper is **72**.
*   The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

*   Do not send this question paper for marking; it should be retained in the centre or destroyed.

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**Turn over**

**1** Simplify $\dfrac{x^4 - 10x^2 + 9}{(x^2 - 2x - 3)(x^2 + 8x + 15)}$. **[4]**

**2** Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. **[3]**

**3** **(i)** Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$, and show that the remainder is $x$. **[4]**

**(ii)** Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3}\,\mathrm{d}x.$$ **[4]**

**4** Use the substitution $x = \frac{1}{3}\sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{6}} \frac{1}{\left(1 - 9x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x.$$ **[6]**

**5** The lines $l_1$ and $l_2$ have equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} + s \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

respectively.

**(i)** Show that $l_1$ and $l_2$ are skew. **[3]**

**(ii)** Find the acute angle between $l_1$ and $l_2$. **[4]**

**(iii)** The point $A$ lies on $l_1$ and $OA$ is perpendicular to $l_1$, where $O$ is the origin. Find the position vector of $A$. **[3]**

**6** Find the coefficient of $x^2$ in the expansion in ascending powers of $x$ of

$$\sqrt{\frac{1 + ax}{4 - x}},$$

giving your answer in terms of $a$. **[8]**

**7** The gradient of a curve at the point $(x, y)$, where $x > -2$, is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y^2(x + 2)}.$$

The points $(1, 2)$ and $(q, 1.5)$ lie on the curve. Find the value of $q$, giving your answer correct to 3 significant figures. **[7]**

**8** A curve has parametric equations

$$x = \frac{1}{t+1}, \qquad y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

    **(i)** Show that the value of $t$ at one of these points is $-2$ and find the value of $t$ at the other point. **[2]**

  **(ii)** Find the equation of the normal to the curve at the point for which $t = -2$. **[6]**

 **(iii)** Find the value of $t$ at the point where this normal meets the curve again. **[2]**

 **(iv)** Find a cartesian equation of the curve, giving your answer in the form $y = f(x)$. **[3]**


**9**     **(i)** Show that $\dfrac{d}{dx}(x \ln x - x) = \ln x$. **[3]**

   **(ii)**
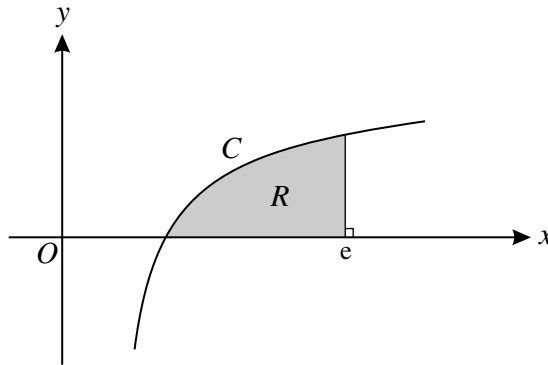


In the diagram, $C$ is the curve $y = \ln x$. The region $R$ is bounded by $C$, the $x$-axis and the line $x = e$.

  **(a)** Find the exact volume of the solid of revolution formed by rotating $R$ completely about the $x$-axis. **[6]**

  **(b)** The region $R$ is rotated completely about the $y$-axis. Explain why the volume of the solid of revolution formed is given by

$$\pi e^2 - \pi \int_0^1 e^{2y}\, dy,$$

and find this volume. **[4]**

**4**