RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4724:** Core Mathematics 4

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

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1	Attempt to factorise **both** numerator & denominator	M1		completely or partially
	Num = e.g. $(x^2-1)(x^2-9)$ or $(x^2-2x-3)(x^2+2x-3)$	B1		or $(x-3)(x+3)(x-1)(x+1)$
	Denominator = e.g. $(x^2-2x-3)(x+5)(x+3)$	B1		or $(x-3)(x+1)(x+5)(x+3)$
	$\frac{x-1}{x+5}$ or $1-\frac{6}{x+5}$ WWW	A1	4	ISW but not if any further 'cancellation'
	<u>Alternative start, attempting long division</u>			
	Expand denom as quartic & attempt to divide $\frac{\text{numerator}}{\text{denominator}}$	M1		but <u>not</u> divide $\frac{\text{denominator}}{\text{numerator}}$
	Obtain quotient = 1 & remainder = $-6x^3-6x^2+54x+54$	B1		
	Final B1 A1 available as before			
				$\boxed{4}$
2	$2^2+(-3)^2+\left(\sqrt{12}\right)^2$ soi e.g. 25 or 5	M1		Allow $2^2-3^2+\sqrt{12}^2$
	5	A1		May be implied by 5 or 1/5 in final answer
	$\frac{1}{5}\begin{pmatrix}2\\-3\\\sqrt{12}\end{pmatrix}$ or $\begin{pmatrix}\frac{2}{5}\\-\frac{3}{5}\\\frac{\sqrt{12}}{5}\end{pmatrix}$ AEF	√A1	3	FT their '5'. Accept $-\frac{1}{5}\begin{pmatrix}\\\\\end{pmatrix}$ or $\frac{1}{\pm 5}\begin{pmatrix}\\\\\end{pmatrix}$
				$\boxed{3}$
3	**(i)** <u>The words quotient and remainder need not be explicit</u>			
	<u>Long division</u> For leading term $3x$ in quotient	B1		
	Suff evidence of div process ($3x$, mult back, attempt sub)	M1		
	(Quotient) = $3x-1$	A1		
	(Remainder) = x **AG**	A1	4	No wrong working, partic on penult line
	<u>Identity</u> $3x^3-x^2+10x-3=Q\left(x^2+3\right)+R$	*M1		
	$Q=ax+b, R=cx+d$ & attempt at least 2 operations	dep*M1		If $a=3$, this $\Rightarrow$ 1 operation
	$a=3, b=-1$	A1		
	$c=1, d=0$	A1		No wrong working anywhere
	<u>Inspection</u> $3x^3-x^2+10x-3=\left(x^2+3\right)(3x-1)+x$	B2		or state quotient $=3x-1$
	Clear demonstration of LHS = RHS	B2		
(ii)	Change integrand to 'their (i) quotient' + $\frac{x}{x^2+3}$	M1		
	Correct FT integration of 'their (i) quotient'	√A1		
	$\int \frac{x}{x^2+3}\,dx=\frac{1}{2}\ln\left(x^2+3\right)$	A1		
	Exact value of integral = $\frac{1}{2}+\frac{1}{2}\ln 4-\frac{1}{2}\ln 3$ AEF ISW	A1	4	Answer as decimal value (only) $\rightarrow$ A0
				$\boxed{8}$

4		<u>Indefinite integral</u> Attempt to connect dx and $d\theta$	M1	Incl $\frac{dx}{d\theta}=, \frac{d\theta}{dx}=$, $dx = ...d\theta$; not $dx = d\theta$
		Denominator $\left(1-9x^2\right)^{\frac{3}{2}}$ becomes $\cos^3\theta$	B1	
		Reduce original integral to $\frac{1}{3}\int \frac{1}{\cos^2\theta}\,d\theta$	A1	May be implied, seen only as $\frac{1}{3}\int \sec^2\theta\,d\theta$
		Change $\int \frac{1}{\cos^2\theta}\,d\theta$ to $\tan\theta$	B1	Ignore $\frac{1}{3}$ at this stage
		Use <u>appropriate</u> limits for θ (allow degrees) or x	M1	Integration need not be accurate
		$\frac{\sqrt{3}}{9}$ AEF, exact answer required, ISW	A1 **6**	
				$\boxed{\textbf{6}}$
5	**(i)**	Attempt to set up 3 equations	M1	of type $4+3s=1, 6+2s=t, 4+s=-t$
		$(s,t)=(-1,4)$ or $(-1,-3)$ or $(-\frac{10}{3},-\frac{2}{3})$	*A1	or $s=-1$ & $-\frac{10}{3}$ <u>or</u> $t=$ two of $\left(4,-3,-\frac{2}{3}\right)$
		Show clear contradiction e.g. $3\neq -4,\ 4\neq -3,\ -6\neq 1$	dep*A1 **3**	Allow ✓ unsimpl contradictions. No ISW.
		<u>SC</u> If $s=\frac{-10}{3}$ found from 2nd & 3rd eqns and contradiction shown in 1st eqn, all 3 marks may be awarded.		
	(ii)	Work with $\begin{pmatrix}3\\2\\1\end{pmatrix}$ and $\begin{pmatrix}0\\1\\-1\end{pmatrix}$	M1	
		Clear method for scalar product of any 2 vectors	M1	
		Clear method for modulus of any vector	M1	
		$79.1^{(\circ)}$ or better (79.1066..) 1.38 (rad) (1.38067..) ISW	A1 **4**	**(**From $\frac{1}{\sqrt{14}.\sqrt{2}}$)
	(iii)	Use $\begin{pmatrix}4+3s\\6+2s\\4+s\end{pmatrix}.\begin{pmatrix}3\\2\\1\end{pmatrix}=0$	M1	
		Obtain $s=-2$	A1	from $12+9s+12+4s+4+s=0$
		A is $\begin{pmatrix}-2\\2\\2\end{pmatrix}$ or $-2\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ final answer	<u>B</u>1 **3**	Accept $(-2,2,2)$
				$\boxed{\textbf{10}}$

6 $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}ax$ …….. $+\frac{\frac{1}{2}\cdot\frac{-1}{2}}{2}(ax)^2$ B1, B1 N.B. third term = $-\frac{1}{8}a^2x^2$

Change $(4-x)^{-\frac{1}{2}}$ into $k\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$, where k is likely to be $\frac{1}{2}$ /2/4 /–2, & work out expansion of $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1+\frac{1}{8}x$ …. $+\frac{\frac{-1}{2}\cdot\frac{-3}{2}}{2}\left(\frac{(-)x}{4}\right)^2$ B1, B1 N.B. third term = $\frac{3}{128}x^2$

<u>OR</u> Change $\{4-x\}^{\frac{1}{2}}$ into $l\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$, where l is likely to be $\frac{1}{2}$ / 2 / 4 /– 2,& work out expansion of $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$

$\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 1-\frac{1}{8}x-\frac{1}{128}x^2$ B1 (for all 3 terms simplified)

$k=\frac{1}{2}$ (with possibility of M1 + A1 + A1 to follow) B1 $l=2$ (<u>with no further marks available</u>)

Multiply $(1+ax)^{\frac{1}{2}}$ by $(4-x)^{-\frac{1}{2}}$ or $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ M1 Ignore irrelevant products

The required three terms (with/without x^2) <u>identified as</u>

$-\frac{1}{16}a^2+\frac{1}{32}a+\frac{3}{256}$ or $\frac{-16a^2+8a+3}{256}$ AEF ISW A1+A1 **8** A1 for one correct term + A1 for other two

<u>**SC**</u> B1 for $\frac{1}{4}\left(1-\frac{x}{4}\right)^{-1}$; B1 for $\left(1-\frac{x}{4}\right)^{-1} = 1+\frac{x}{4}+\frac{x^2}{16}$; M1 for multiplying $(1+ax)$ by their $(4-x)^{-1}$.

If result is $p+qx+rx^2$, then to find $\left(p+qx+rx^2\right)^{\frac{1}{2}}$ award B1 for $p^{\frac{1}{2}}(\ldots\ldots)$,

B1 correct 1st & 2nd terms of expansion, B1 correct 3rd term; A1,A1 as before, for correct answers.

$\boxed{8}$

7 Attempt to sep variables in format $\int py^2\,(\mathrm{d}y) = \int \frac{q}{x+2}(\mathrm{d}x)$ M1 where constants p and/or q may be wrong

Either y^3 & $\ln(x+2)$ <u>or</u> $\frac{1}{3}y^3$ & $\frac{1}{3}\ln(x+2)$ A1+A1 Accept $\frac{1}{3}\ln(3x+6)$ for $\frac{1}{3}\ln(x+2)$ & $|\ |$ for ( )

<u>If indefinite integrals are being used (most likely scenario)</u>

Substitute $x=1, y=2$ into an eqn <u>containing '+const'</u> M1

Sub <u>y</u> = 1.5 and their value of 'const' & solve for <u>*x or q*</u> M1

x or q = –1.97 <u>only</u> A2

[SC x or q = –1.970 or –1.971 or –1.9705 or –1.9706 A1] **7**

<u>If definite integrals are used (less likely scenario)</u>

Use $\int_{1.5}^{2}\ldots\mathrm{d}y = \int_{q}^{1}\ldots\mathrm{d}x$ where 2 corresponds with 1..... M2 & 1.5 corresp with q (at top/bottom or v.v.)

Then A2 or SC A1 as above

Use $\int_{1.5}^{2}\ldots\mathrm{d}y = \int_{1}^{q}\ldots\mathrm{d}x$ where 2 corresponds with q….. M1 & 1.5 corresp with 1 (at top/bottom or v.v.)

Then A1 for 1.97 <u>only</u>

$\boxed{7}$

8	**Cartesian equation may be used in parts (i) - (iii) and corresponding marks awarded**			
(i)	Sub parametric eqns into $y = 3x$ & produce $t = -2$ OR sub $t = -2$ into para eqs, obtain $(-1, -3)$ & state $y = 3x$ OR other similar methods producing (or verifying) $t = -2$	B1		
	Value of t at other point is 2	B1	**2**	$t = \pm 2$ is sufficient for B1+B1
(ii)	Use (not just quote) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		
	$= -(t+1)^2$	A1		or $\frac{-1}{x^2}$ or $\frac{-(2+y)}{x}$
	Attempt to use $-\frac{1}{\frac{dy}{dx}}$ for gradient of normal	M1		
	Gradient normal = 1 cao	A1		
	Subst $t = -2$ into the parametric eqns.	M1		to find pt at which normal is drawn
	Produce $y = x - 2$ as equation of the normal WWW	A1	**6**	'A' marks in **(ii)** are dep on prev 'A'
(iii)	Substitute the parametric values into their eqn of normal	M1		
	Produce $t = 0$ as final answer cao	A1	**2**	This is dep on final A1 in (ii)
	N.B. If $y = x - 2$ is found fortuitously in (ii) (& ∴ given A0 in (ii)), you must award A0 here in (iii).			
(iv)	Attempt to eliminate t from the parametric equations	M1		
	Produce any correct equation	A1		e.g. $x = \frac{1}{y+2}$
	Produce $y = \frac{1}{x} - 2$ or $y = \frac{1-2x}{x}$ ISW	A1	**3**	Must be seen in (iv)
	{N.B. Candidate producing only $y = \frac{1}{x} - 2$ is awarded both A1 marks.}			
			$\boxed{13}$	

9 (i)	Treat $x \ln x$ as a product	M1		If $\int \ln x$, use parts $u = \ln x$, $dv = 1$
	Obtain $x.\frac{1}{x} + \ln x$	A1		$x \ln x - \int 1\,dx = x \ln x - x$
	Show $x.\frac{1}{x} + \ln x - 1 = \ln x$ WWW **AG**	A1	**3**	And state given result
(ii)(a)	Part (a) is mainly based on the indef integral $\int (\ln x)^2\,dx$			
	[A candidate stating e.g. $\int (\ln x)^2\,dx = \int 2\ln x\,dx$ or $= \int (\ln x - x)^2\,dx$ **is awarded 0 for (ii)(a)]**			
	Correct use of $\int \ln x\,dx = x\ln x - x$ anywhere in this part	B1		Quoted from (i) or derived
	Use integ by parts on $\int (\ln x)^2\,dx$ with $u = \ln x, dv = \ln x$	M1		or $u = (\ln x)^2, dv = 1$
	[For 'integration by parts, candidates must get to a 1st stage with format $f(x) +/- \int g(x)\,dx$]			
	1st stage $= \ln x(x\ln x - x) - \int \frac{1}{x}(x\ln x - x)\,dx$ soi	A1		$x(\ln x)^2 - \int x.\frac{2}{x}\ln x\,dx$
	2nd stage $= x(\ln x)^2 - 2x\ln x + 2x$ AEF (unsimplified)	A1		
	$\therefore$ Value of definite integral between 1 & e $= e - 2$ cao	A1		Use limits on 2nd stage & produce cao
	Volume $= \pi(e - 2)$ ISW	A1	**6**	Answer as decimal value (only) $\rightarrow$ A0
	Alternative method when subst. $u = \ln x$ used			
	Attempt to connect dx and du	M1		
	Becomes $\int u^2 e^u\,du$	A1		
	First stage $u^2e^u - \int 2u\,e^u\,du$	A1		
	Third stage $(u^2 - 2u + 2)e^u$	A1		
	Final A1 A1 available as before			
(b)	Indication that reqd vol = vol cylinder – vol inner solid	M1		
	Clear demonstration of either vol of cylinder being πe^2 (including reason for height $= \ln e$) or rotation of $x = e$ about the y-axis (including upper limit of $y = \ln e$)	A1		Could appear as $\pi\int_0^1 e^2\,dy$
	$(\pi)\int x^2\,dy = (\pi)\int e^{2y}\,dy$	B1		
	$\frac{\pi(e^2+1)}{2}$ or 13.2 or 13.18 or better	B1	**4**	May be from graphical calculator
			13	

Possible helpful points

1. M is Method; does the candidate know what he/she should be doing? It does not ask how accurate it is.. e.g. in Qu.4, a candidate saying $\frac{dx}{d\theta} = -\frac{1}{3}\cos\theta$ is awarded M1.
2. When checking if decimal places are acceptable, accept both rounding & truncation.
3. In general we ISW unless otherwise stated.
4. The symbol √ is sometimes used to indicate 'follow-through' in this scheme.

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