OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4724

Core Mathematics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Expand $(1+3x)^{-\frac{5}{3}}$ in ascending powers of x, up to and including the term in x^3. **[5]**

2 Given that $y = \dfrac{\cos x}{1 - \sin x}$, find $\dfrac{dy}{dx}$, simplifying your answer. **[4]**

3 Express $\dfrac{x^2}{(x-1)^2(x-2)}$ in partial fractions. **[5]**

4 Use the substitution $u = \sqrt{x+2}$ to find the exact value of

$$\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}\,dx.$$ **[7]**

5 Find the coordinates of the two stationary points on the curve with equation

$$x^2 + 4xy + 2y^2 + 18 = 0.$$ **[7]**

6 Lines l_1 and l_2 have vector equations

$$\mathbf{r} = \mathbf{j} + \mathbf{k} + t(2\mathbf{i} + a\mathbf{j} + \mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 3\mathbf{i} - \mathbf{k} + s(2\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$$

respectively, where t and s are parameters and a is a constant.

(i) Given that l_1 and l_2 are perpendicular, find the value of a. **[3]**

(ii) Given instead that l_1 and l_2 intersect, find

(a) the value of a, **[4]**

(b) the angle between the lines. **[3]**

7 The parametric equations of a curve are $x = \dfrac{t+2}{t+1}$, $y = \dfrac{2}{t+3}$.

(i) Show that $\dfrac{dy}{dx} > 0$. **[6]**

(ii) Find the cartesian equation of the curve, giving your answer in a form not involving fractions. **[5]**

8 **(i)** Find the quotient and the remainder when $x^2 - 5x + 6$ is divided by $x - 1$. **[3]**

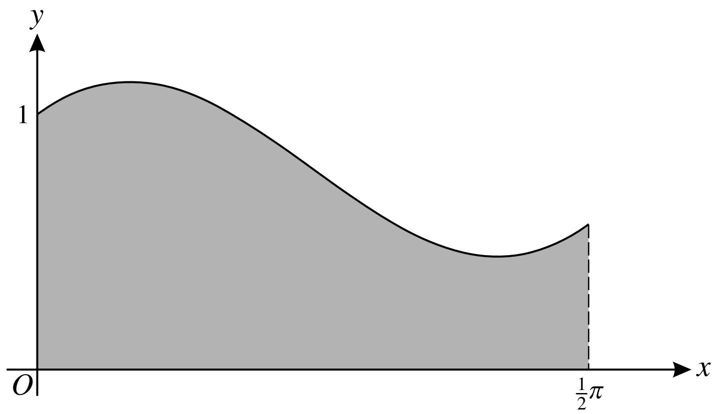
(ii) **(a)** Find the general solution of the differential equation

$$\left(\frac{x-1}{x^2-5x+6}\right)\frac{dy}{dx} = y - 5.$$ **[3]**

(b) Given that $y = 7$ when $x = 8$, find y when $x = 6$. **[4]**

9 **(i)** Find $\int (x + \cos 2x)^2 \, dx$. **[9]**

(ii)



The diagram shows the part of the curve $y = x + \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The shaded region bounded by the curve, the axes and the line $x = \frac{1}{2}\pi$ is rotated completely about the x-axis to form a solid of revolution of volume V. Find V, giving your answer in an exact form. **[4]**