C4 June 2010

① $$(1+3x)^{-5/3} = 1 + -\frac{5}{3}\times 3x + \frac{\frac{-5}{3}\times\frac{-8}{3}(3x)^2}{2!} + \frac{\frac{-5}{3}\times\frac{-8}{3}\times\frac{-11}{3}(3x)^3}{3!}$$

$$= 1 - 5x + 20x^2 - \frac{220}{3}x^3$$

② $$\frac{dy}{dx} = \frac{(1-\sin x)\times -\sin x - \cos x \times -\cos x}{(1-\sin x)^2}$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2} = \frac{1-\sin x}{(1-\sin x)^2} = \frac{1}{1-\sin x}$$

③ $$= \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$$

$$\Rightarrow x^2 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

$x=1 \Rightarrow \quad 1 = -B \Rightarrow B = -1$

$x=2 \Rightarrow \quad 4 = C$

$x^2: \Rightarrow \quad 1 = A + C \Rightarrow A = -3$

$$\Rightarrow \quad \frac{-3}{x-1} - \frac{1}{(x-1)^2} + \frac{4}{x-2}$$

④ $u = \sqrt{x+2} \Rightarrow u^2 = x+2 \Rightarrow x = u^2 - 2 \Rightarrow dx = 2u\,du$.

$x = 7 \Rightarrow u = 3, \quad x = -1 \Rightarrow u = 1$

$$I = \int_1^3 \frac{(u^2-2)^2\, 2u\,du}{u} = 2\int_1^3 u^4 - 4u^2 + 4\,du$$

$$= 2\left[\frac{u^5}{5} - \frac{4u^3}{3} + 4u\right]_1^3 = 2\left(48.6 - 36 + 12 - \left(\frac{1}{5} - \frac{4}{3} + 4\right)\right)$$

$$= \frac{652}{15}$$

⑤ $$2x + 4x\frac{dy}{dx} + 4y + 4y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x-4y}{4x+4y} = 0 \text{ at SP}$$

$$\Rightarrow y = -\frac{1}{2}x$$

Sub in $\Rightarrow x^2 - 2x^2 + \frac{1}{2}x^2 + 18 = 0$

$\Rightarrow x = \pm 6.$

SP $(6, -3)$ and $(-6, 3)$

⑥ (i) $\perp \Rightarrow \begin{pmatrix} 2 \\ a \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix} = 0 \Rightarrow 4 + 2a - 6 = 0$

$\Rightarrow a = 1$

(ii) (a) $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ a \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$

$2t - 2s = 3$

$at - 2s = -1$

$t + 6s = -2$

$\} \; 14s = -7 \Rightarrow s = -\frac{1}{2}$

$t = 1$

$at - 2s = -1 \Rightarrow a + 1 = -1 \Rightarrow a = -2$

(b) $\cos\theta = \dfrac{\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}}{\sqrt{9 \times 44}} = \dfrac{-6}{19.899} \Rightarrow \theta = 108^\circ$

Acute angle $= \underline{72.5^\circ}$

⑦ (i) $\dfrac{dx}{dt} = \dfrac{(t+1)\times 1 - (t+2)\times 1}{(t+1)^2} = \dfrac{-1}{(t+1)^2}$

$\dfrac{dy}{dt} = \dfrac{-2}{(t+3)^2}$

$\Big\} \; \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2(t+1)^2}{(t+3)^2}$

$\dfrac{dy}{dx} = 2\left(\dfrac{t+1}{t+3}\right)^2$ which is a square number so cannot be negative. Nor can $\dfrac{dy}{dx} = 0$ because $\dfrac{dy}{dx} = 0 \Rightarrow t = -1$ and x is undefined if $t = -1$

so $\dfrac{dy}{dx} > 0$

(ii) $y = \dfrac{2}{t+3} \Rightarrow t = \dfrac{2}{y} - 3$

$x = \dfrac{t+2}{t+1} = \dfrac{2/y - 3 + 2}{2/y - 3 + 1} = \dfrac{2/y - 1}{2/y - 2} = \dfrac{2-y}{2-2y}$

$(2-2y)x = 2-y$

⑧ (i)

$$\begin{array}{r|l} & x-4 \\ x-1 & x^2-5x+6 \\ & x^2-x \\ \hline & -4x+6 \\ & -4x+4 \\ \hline & R=2 \end{array}$$

Quotient $x-4$

Remainder 2 or $\frac{2}{x-1}$

(ii) Separate the Variables, $\Rightarrow \int \frac{1}{y-5}\,dy = \int \frac{x^2-5x+6}{x-1}\,dx$

$$\Rightarrow \int \frac{1}{y-5}\,dy = \int x-4 + \frac{2}{x-1}\,dx.$$

$$\Rightarrow \ln(y-5) = \frac{x^2}{2} - 4x + 2\ln(x-1) + c.$$

or, $\ln \frac{y-5}{(x-1)^2} = \frac{x^2}{2} - 4x + c$

$$\Rightarrow \frac{y-5}{(x-1)^2} = Ae^{\frac{x^2}{2}-4x} \quad \text{where } A = e^c$$

(b) $y=7,\ x=8 \Rightarrow \frac{2}{49} = Ae^0 \Rightarrow A = 2/49$

When $x=6$, $\frac{y-5}{25} = \frac{2}{49}e^{-6}$

$$y = \frac{50}{49}e^{-6} + 5 = 5.0025 \ (5SF)$$

⑨ (i) $= \int x^2 + 2x\cos 2x + \cos^2 2x \,dx$

$$= \frac{x^3}{3} + x\sin 2x - \int \sin 2x \cdot 1\,dx + \int \frac{1}{2}(1+\cos 4x)\,dx.$$

$$= \frac{x^3}{3} + x\sin 2x + \frac{1}{2}\cos 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x + c.$$

$$V = \pi\int_0^{\pi/2} (x+\cos 2x)^2\,dx = \pi\left[\frac{x^3}{3} + x\sin 2x + \frac{1}{2}\cos 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x\right]_0^{\pi/2}$$

$$= \pi\left[\frac{\pi^3}{3\times 8} + 0 - \frac{1}{2} + \frac{\pi}{4} + 0 - \left(0 + 0 + \frac{1}{2} + 0 + 0\right)\right]$$

$$= \frac{\pi^4}{24} + \frac{\pi^2}{4} - \pi$$