



**ADVANCED GCE UNIT  
MATHEMATICS**

Core Mathematics 4  
**THURSDAY 14 JUNE 2007**

**4724/01**

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)  
List of Formulae (MF1)

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

**ADVICE TO CANDIDATES**

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The equation of a curve is  $y = f(x)$ , where  $f(x) = \frac{3x + 1}{(x + 2)(x - 3)}$ .

(i) Express  $f(x)$  in partial fractions. [2]

(ii) Hence find  $f'(x)$  and deduce that the gradient of the curve is negative at all points on the curve. [3]

2 Find the exact value of  $\int_0^1 x^2 e^x dx$ . [6]

3 Find the exact volume generated when the region enclosed between the  $x$ -axis and the portion of the curve  $y = \sin x$  between  $x = 0$  and  $x = \pi$  is rotated completely about the  $x$ -axis. [6]

4 (i) Expand  $(2 + x)^{-2}$  in ascending powers of  $x$  up to and including the term in  $x^3$ , and state the set of values of  $x$  for which the expansion is valid. [5]

(ii) Hence find the coefficient of  $x^3$  in the expansion of  $\frac{1 + x^2}{(2 + x)^2}$ . [2]

5 A curve  $C$  has parametric equations

$$x = \cos t, \quad y = 3 + 2 \cos 2t, \quad \text{where } 0 \leq t \leq \pi.$$

(i) Express  $\frac{dy}{dx}$  in terms of  $t$  and hence show that the gradient at any point on  $C$  cannot exceed 8. [4]

(ii) Show that all points on  $C$  satisfy the cartesian equation  $y = 4x^2 + 1$ . [3]

(iii) Sketch the curve  $y = 4x^2 + 1$  and indicate on your sketch the part which represents  $C$ . [2]

6 The equation of a curve is  $x^2 + 3xy + 4y^2 = 58$ . Find the equation of the normal at the point  $(2, 3)$  on the curve, giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. [8]

7 (i) Find the quotient and the remainder when  $2x^3 + 3x^2 + 9x + 12$  is divided by  $x^2 + 4$ . [4]

(ii) Hence express  $\frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}$  in the form  $Ax + B + \frac{Cx + D}{x^2 + 4}$ , where the values of the constants  $A$ ,  $B$ ,  $C$  and  $D$  are to be stated. [1]

(iii) Use the result of part (ii) to find the exact value of  $\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4} dx$ . [5]

- 8 The height,  $h$  metres, of a shrub  $t$  years after planting is given by the differential equation

$$\frac{dh}{dt} = \frac{6-h}{20}.$$

A shrub is planted when its height is 1 m.

- (i) Show by integration that  $t = 20 \ln\left(\frac{5}{6-h}\right)$ . [6]
- (ii) How long after planting will the shrub reach a height of 2 m? [1]
- (iii) Find the height of the shrub 10 years after planting. [2]
- (iv) State the maximum possible height of the shrub. [1]
- 9 Lines  $L_1$ ,  $L_2$  and  $L_3$  have vector equations

$$L_1: \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}),$$

$$L_2: \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}),$$

$$L_3: \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + u(3\mathbf{i} + c\mathbf{j} + \mathbf{k}).$$

- (i) Calculate the acute angle between  $L_1$  and  $L_2$ . [4]
- (ii) Given that  $L_1$  and  $L_3$  are parallel, find the value of  $c$ . [2]
- (iii) Given instead that  $L_2$  and  $L_3$  intersect, find the value of  $c$ . [5]

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