RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4724:** Core Mathematics 4

Mark Scheme for January 2011

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Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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1	**(i)**	First two terms are $1-\frac{1}{2}x$........	B1	
		Third term = $\frac{\frac{1}{2}.-\frac{1}{2}}{2}[(-x)^2$ or x^2 or $-x^2]$	M1	
		$= -\frac{1}{8}x^2$	A1 **3**	$-\frac{1}{8}x^2$ without work $\rightarrow$ M1 A1
	(ii)	Attempt to replace x by $2y-4y^2$ or $2y+4y^2$	M1	or write as $1-(2y-4y^2$ or $2y+4y^2)$
		First two terms are $1-y$	B1	
		Third term = $+\frac{3}{2}y^2$ or $\surd$ $(4b+2)y^2$	A1√ **3**	where b = cf(x^2) in part (i)
			6	
2	**(i)**	$A(x-2)+B=7-2x$	M1	or $A(x-2)^2+B(x-2)=(7-2x)(x-2)$
		$A=-2$	A1	
		$B=3$	A1 **3**	
	(ii)	$\int \frac{A}{x-2}\,dx = \left(A \text{ or } \frac{1}{A}\right)\ln(x-2)$	B1	Accept $\ln\|x-2\|$, $\ln\|2-x\|$, $\ln(2-x)$
		$\int \frac{B}{(x-2)^2}\,dx = -\left(B \text{ or } \frac{1}{B}\right).\frac{1}{x-2}$	B1	Negative sign <u>is</u> required
		Correct f.t. of A & B; $A\ln(x-2)-\frac{B}{x-2}$	B1√	Still accept lns as before
		Using limits = $-2\ln 3+2\ln 2+\frac{1}{2}$ ISW	B1 **4**	<u>No</u> indication of ln(negative)
			7	
3	**(i)**	State/imply $\frac{d}{dx}(\sec x)=\frac{d}{dx}\left(\frac{1}{\cos x}\right)$ or $\frac{d}{dx}(\cos x)^{-1}$	B1	Not just $\sec x = \frac{1}{\cos x}$
		Attempt quotient rule or chain rule to power -1	M1	Allow $\frac{u\,dv - v\,du}{v^2}$ & wrong trig signs
		Obtain $\frac{\sin x}{\cos^2 x}$ or $-.-(\sin x)(\cos x)^{-2}$	A1	No inaccuracy allowed here
		Simplify with suff evid to **AG** e.g. $\frac{1}{\cos x}.\frac{\sin x}{\cos x}$	A1 **4**	Or vice versa. <u>Not</u> just = sec x.tan x
	(ii)	Use $\cos 2x = +/-1+/-2\cos^2 x$ or $+/-1+/-2\sin^2 x$	M1	or $\pm(\cos^2 x-\sin^2 x)$
		Correct denominator = $\sqrt{2\cos^2 x}$	A1	$\sqrt{2-2\sin^2 x}$ needs simplifying
		Evidence that $\frac{\tan x}{\cos x}$ = sec x tan x or $\int \frac{\tan x}{\cos x}dx = \sec x$	B1	irrespective of any const multiples
		$\frac{1}{\sqrt{2}}\sec x$ (+ c)	A1 **4**	Condone θ for x except final line
			8	

4	**(i)**	Attempt to use $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{dy}{dt}.\frac{dt}{dx}$	M1		Not just quote formula
		$\frac{4}{2t}$ or $\frac{2}{t}$	A1	**2**	
	(ii)	Subst $t=4$ into their (i), invert & change sign	M1		
		Subst $t=4$ into (x,y) & use num grad for tgt/normal	M1		
		$y=-2x+52$ AEF CAO (no f.t.)	A1	**3**	Only the eqn of normal accepted
	(iii)	Attempt to eliminate t from the 2 given equations	M1		
		$x=2+\frac{y^2}{16}$ or $y^2=16(x-2)$ AEF ISW	A1	**2**	Mark at earliest acceptable form.
					7
5	**(i)**	Attempt to connect dx and du	M1		Including $\frac{du}{dx}=$ or $du=...dx$; not dx = du
		$5-x=4-u^2$	B1		perhaps in conjunction with next line
		Show $\int \frac{4-u^2}{2+u}.2u\ du$ reduced to $\int 4u-2u^2\ du$ **AG**	A1		In a fully satisfactory & acceptable manner
		Clear explanation of why limits change	B1		e.g. when x = 2, u = 1 <u>and</u> when x = 5, u = 2
		$\frac{4}{3}$	B1	**5**	not dependent on any of first 4 marks
	(ii)(a)	$5-x$	*B1	**1**	Accept $4-x-1=5-x$ (this is not **AG**)
	(b)	Show reduction to $2-\sqrt{x-1}$	dep*B1		
		$\int\sqrt{x-1}\,dx=\frac{2}{3}(x-1)^{\frac{3}{2}}$	B1		Indep of other marks, seen anywhere in (b)
		$\left(10-\frac{2}{3}.8\right)-\left(4-\frac{2}{3}\right)=\frac{4}{3}$ or $4\frac{2}{3}-3\frac{1}{3}=\frac{4}{3}$	B1	**3**	Working must be shown
					9
6	**(i)**	Work with correct pair of direction vectors	M1		
		Demonstrate correct <u>method</u> for finding scalar product	M1		Of <u>any</u> two 3x3 vectors rel to question
		Demonstrate correct <u>method</u> for finding modulus	M1		Of <u>any</u> vector relevant to question
		24, 24.0 (24.006363..) (degrees) 0.419 (0.41899..) (rad)	A1	**4**	Mark earliest value, allow trunc/rounding
	(ii)	Attempt to set up 3 equations	M1		Of type $3+2s=5, 3s=3+t, -2-4s=2-2t$
		Find correct values of $(s,t)=(1,0)$ or $(1,4)$ or $(5,12)$	A1		Or 2 diff values of s (or of t)
		Substitute their (s,t) into equation not used	M1		and make a relevant deduction
		<u>Correctly</u> demonstrate failure	A1	**4**	dep on all 3 prev marks
	(iii)	Subst their (s,t) from first 2 eqns into new 3rd eqn	M1		New 3rd eqn of type $a-4s=2-2t$
		$a=6$	A1	**2**	
					10

7		Attempt parts with $u = x^2+5x+7$, $dv = \sin x$	M1		as far as $f(x) +/- \int g(x)\,dx$
		1st stage = $-(x^2+5x+7)\cos x + \int (2x+5)\cos x\,dx$	A1		signs need not be amalgamated at this stage
		$\int (2x+5)\cos x\,dx = (2x+5)\sin x - \int 2\sin x\,dx$	B1		indep of previous A1 being awarded
		$= (2x+5)\sin x + 2\cos x$	B1		
		I = $-(x^2+5x+7)\cos x + (2x+5)\sin x + 2\cos x$	A1		WWW
		(Substitute $x=\pi$) $-$(Substitute $x=0$)	M1		An attempt at subst $x = 0$ must be seen
		$\pi^2+5\pi+10$ WWW **AG**	A1	**7**	
					7
8	(i)	$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
		$\frac{d}{dx}(-5xy) = (-)(5)x\frac{dy}{dx} + (-)(5)y$	M1		i.e. reasonably clear use of product rule
		LHS completely correct $4x - 5x\frac{dy}{dx} - 5y + 2y\frac{dy}{dx} (= 0)$	A1		Accept " $\frac{dy}{dx} =$ " provided it is not used
		Substitute $\frac{dy}{dx} = \frac{3}{8}$ or solve for $\frac{dy}{dx}$ & then equate to $\frac{3}{8}$	M1		Accuracy not required for "solve for $\frac{dy}{dx}$ "
		Produce $x = 2y$ WWW **AG** **(**Converse acceptable)	A1	**5**	Expect $17x = 34y$ and/or $\frac{dy}{dx} = \frac{5y-4x}{2y-5x}$
	(ii)	Substitute $2y$ for x or $\frac{1}{2}x$ for y in curve equation	M1		
		Produce either $x^2 = 36$ or $y^2 = 9$	A1		
		AEF of $(\pm 6, \pm 3)$	A1	**3**	ISW Any correct format acceptable
					8
9	(i)	Attempt to sep variables in the form $\int \frac{p}{(x-8)^{1/3}}\,dx = \int q\,dt$	M1		Or invert as $\frac{dt}{dx} = \frac{r}{(x-8)^{1/3}}$; p,q,r consts
		$\int \frac{1}{(x-8)^{1/3}}\,dx = k(x-8)^{2/3}$	A1		k const
		All correct (+ c)	A1		
		For equation containing 'c'; substitute $t = 0$, $x = 72$	M1		M2 for $\int_{72}^{35} = \int_0^t$ or $\int_{35}^{72} = \int_0^t$
		Correct corresponding value of c from correct eqn	A1		
		Subst their c & $x = 35$ back into eqn	M1		
		$t = \frac{21}{8}$ or 2.63 / 2.625 [C.A.O]	A1	**7**	A2: $t = \frac{21}{8}$ or 2.63 / 2.625 WWW
	(ii)	State/imply in some way that $x = 8$ when flow stops	B1		
		Substitute $x = 8$ back into equation containing numeric 'c'	M1		
		$t = 6$	A1	**3**	
					10

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